Essays in Quantitative Macroeconomics: Income, Inequality, Income Risk and Optimal Redistribution

Philipp Grübener

Thesis submitted for assessment with a view to obtaining the degree of Doctor of Economics of the European University Institute

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Department of Economics

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24.10.2021

Abstract

This thesis contains four independent essays in heterogeneous agent macroeconomics. They explore the sources of income inequality and income risk and study the optimal design of public redistribution and insurance.

The first chapter, joint with Filip Rozsypal, studies the origins of idiosyncratic earnings risk in frictional labor markets, with a particular focus on the role of firms for worker earnings risk. First, using administrative matched employer-employee data from Denmark, we document key properties of the worker earnings growth distribution, the firm revenue growth distribution, and their joint distribution. The worker earnings and firm revenue growth distributions exhibit strong deviations from normality, in particular excess kurtosis, with many workers and firms experiencing very small changes to their earnings/revenues, but a significant minority experiencing very large changes. Large earnings losses are more likely for workers in firms with negative revenue growth, driven both by separations to unemployment and earnings losses on the job. Second, we develop a model framework consistent with the data, with four key features: i) frictional labor markets and on the job search to capture unemployment risk and wage growth through a job ladder, ii) multi-worker firms to capture gross and net worker flows, iii) risk averse workers such that earnings risk matters, and iv) contracting with two-sided limited commitment because earnings of job stayers are changing infrequently in the data. Third, we use the model to explore policies designed to mitigate earnings fluctuations.

The second chapter, joint with Annika Bacher and Lukas Nord, studies one particular private insurance margin against individual income risk only available to couples, which is the so called added worker effect. Specifically, we study how this intra-household insurance against individual job loss through increased spousal labor market participation varies over the life cycle. We show in U.S. data that the added worker effect is much stronger for young than for old households. A stochastic life cycle model of two-member households with job search in a frictional labor market is capable of replicating this finding. The model suggests that a lower added worker effect for the old is driven primarily by better insurance through asset holdings. Human capital differences between employed young and old

contribute to the difference but are quantitatively less important, while differences in job arrival rates play a limited role.

In the third chapter, joint with Axelle Ferriere, Gaston Navarro, and Oliko Vardishvili, we study optimal redistribution, taking into account not just the large income and wealth inequality in the data, but also the distribution of income risk that is key in the first two chapters. The U.S. fiscal system redistributes through a rich set of taxes and transfers, the latter accounting for a large part of the income of the poor. Motivated by this, we study the optimal joint design of transfers and income taxes. Within a simple heterogeneous-household framework, we derive analytical results on the optimal relationship between transfers and tax progressivity. Higher transfers are associated with lower optimal income tax progressivity. Redistribution is achieved with generous transfers while efficiency is preserved via a lower progressivity of income taxes. As such, the optimal tax-and-transfer system features larger progressivity of average than of marginal tax rates. We then quantify the optimal tax-and-transfer system in a rich incomplete-market model with realistic distributions of income, wealth, and income risk. The model features a novel flexible functional form for progressive income taxes and means-tested transfers. Relative to the current U.S. fiscal system, the optimal policy consists of more generous means-tested transfers, which phase-out at a slower rate. These larger transfers are financed with higher tax rates, but the taxes are not more progressive than the current system.

The fourth chapter, joint with Axelle Ferriere and Dominik Sachs, also studies optimal redistribution, but instead of considering a stationary environment it analyzes the dynamics of the equity-efficiency trade-off along the growth path. To do so, we incorporate the optimal income taxation problem into a state-of-the-art multi-sector structural change general equilibrium model with non-homothetic preferences. We identify two key opposing forces. First, long-run productivity growth allows households to shift their consumption expenditures away from necessities. This implies a reduction in the dispersion of marginal utilities, and therefore calls for a welfare state that declines along the growth path. Yet, economic growth is also systematically associated with an increase in the skill premium, which raises inequality and the desire to redistribute. We quantitatively analyze these opposing forces for two countries: the U.S. from 1950 to 2010, and China from 1989 to 2009. Optimal redistribution decreases at early stages of development, as the role of non-homotheticities prevails. At later stages of development the rising income inequality dominates and the welfare state should become more generous.

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Many people contributed to the ideas contained in this thesis. I particularly thank my advisors Árpád Ábrahám and Philipp Kircher for their continuous guidance and support and their constant availability and dedication to their students even after having left the EUI. I would also like to particularly thank Axelle Ferriere, who has been an outstanding teacher, advisor, and coauthor throughout the last years. I also thank my coauthors Annika Bacher, Gaston Navarro, Lukas Nord, Filip Rozsypal, Dominik Sachs, and Oliko Vardishvili. Working with them has made the process of writing this thesis so much more enjoyable. Many thanks to Kjetil Storesletten and Ludo Visschers for generously agreeing to be part of my thesis committee.

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Contents

Chapter 1

Firm Dynamics and Earnings Risk[1]

Abstract We study the sources of individual earnings risk in frictional labor markets, focusing on the role firms play for their workers' earnings risk. First, using administrative matched employer-employee data from Denmark, we document key properties of the worker earnings growth distribution, the firm revenue growth distribution, and their joint distribution. The worker earnings and firm revenue growth distributions exhibit strong deviations from normality, in particular excess kurtosis, with many workers and firms experiencing very small changes to their earnings/revenues, but a significant minority experiencing very large changes. Large earnings losses are more likely for workers in firms with negative revenue growth, driven both by separations to unemployment and earnings losses on the job. Second, we develop a model framework consistent with the data, with four key features: i) frictional labor markets and on the job search to capture unemployment risk and wage growth through a job ladder, ii) multi-worker firms to capture gross and net worker flows, iii) risk averse workers such that earnings risk matters, and iv) contracting with two-sided limited commitment because earnings of job stayers are changing infrequently in the data. Third, we use the model to explore policies designed to mitigate earnings fluctuations.

1.1 Introduction

The welfare costs of idiosyncratic risk faced by individuals are large. Constantinides (2021) estimates that the benefits of eliminating idiosyncratic consumption shocks are up to 50% of household utility. For the majority of households the most important source of income to finance their consumption is

[1]This paper uses data from administrative registers from Denmark collected by Statistics Denmark. It also uses data from the Longitudinal Model (version 1993 – 2014) of the Linked Employer-Employee Data from the IAB. This data was accessed on-site at the Research Data Centre (FDZ) of the German Federal Employment Agency (BA) at the Institute for Employment Research (IAB) and via remote data access at the FDZ.

their labor earnings. In this paper we investigate the determinants of labor income risk in frictional labor markets. What does the labor income process households face look like? How important are unemployment, wages, and hours in explaining earnings fluctuations? How closely are worker earnings related to the performance of the firms individuals are employed in? Which policies can be employed to efficiently reduce labor earnings risk and mitigate costly consumption fluctuations?

To answer these questions, we proceed in two steps. First, we document key features of individual earnings risk using administrative registry data from Denmark. We show that higher order moments of the earnings growth distribution are key for understanding the nature of individual earnings risk, as in Guvenen, Karahan, Ozkan, and Song (2021) for the U.S. While the data for many countries contains information only on total earnings, the detailed Danish registry data allows us to zoom in on the contribution of unemployment, wages, and hours to individual earnings risk. Also, exploiting the matched employer-employee dimension of the data, we explore the link between the performance of firms and the earnings growth distribution of the workers employed at these firms. Second, we develop a model of multi-worker firms operating in frictional labor markets featuring risk averse workers who search on- and off-the-job. We calibrate the model to the Danish data in order to have a framework in which we can think about the design of policies efficiently mitigating earnings fluctuations of households.

We start in the empirical part by documenting key properties of the annual labor earnings growth distribution of workers. As has been shown for many countries in the recent literature on earnings risk, the earnings growth distribution exhibits important deviations from normality, which has been the standard assumption about the distribution of earnings shocks until recently. Under the assumption of Gaussian labor income risk many people experience medium sized earnings changes most of the time. Compared to such a normal distribution the empirical earnings growth distribution contains many more individuals with very small earnings changes, many fewer individuals with medium sized earnings changes, and a significantly fatter tail of individuals who experience very large earnings changes: The earnings growth distribution exhibits excess kurtosis. To better understand the nature of individual income risk, we investigate whether these properties are exclusively driven by unemployment risk. When we restrict ourselves to a sample of workers who are continuously employed and have no unemployment spells, we recover similar properties of the earnings growth distribution. While the dispersion of earnings growth is significantly lower for this subsample because the very largest earnings changes are associated with unemployment spells, the distribution still exhibits excess kurtosis. This is also the case when we consider the hourly wage growth distribution instead of the annual earnings growth distribution for continuously employed individuals.

In the second step of the empirical part we investigate the relation of firms to the earnings growth distribution of their workers. The firm revenue growth distribution has very similar features compared to the earnings growth distribution of workers. There are many firms whose revenues do not change much, but there is also a significant share of firms having large revenue changes from one year to the next. How are these two distributions linked? It is well known that on average there is pass-through from firm-level shocks to worker wages (Guiso, Pistaferri, and Schivardi, 2005). We add to this evidence by taking a broader view in considering the entire distribution of earnings growth and by considering both unemployment risk and wage changes. We show that indeed in the Danish data mean earnings growth of workers who are employed in firms whose revenues go down is significantly lower than mean earnings growth of workers in firms that are growing. There is, however, significant heterogeneity when looking at the entire distribution. When comparing the earnings growth distributions of workers in shrinking and growing firms there is significant overlap. In particular, in both groups there are many workers whose earnings are very stable. However, the probability of large earnings drops is much higher in shrinking firms. Again we zoom in on the role of unemployment, wages, and hours for this pattern. First, workers employed in firms with shrinking revenues are much more likely to experience a transition to unemployment, which is associated with the very largest earnings drops. Second, the fact that large earnings drops are more likely in shrinking firms remains true when restricting the sample to workers who are continuously employed at the same firm without unemployment spell. Third, for these workers this pattern is mostly driven by changes to their wages, not their hours.

Motivated by this evidence, we develop a model of firm and worker dynamics in a frictional labor market. Key model ingredients are search on- and off-the-job, multi-worker firms, risk averse workers, and meaningful contracting between workers and firms. Frictional labor markets are key to capture unemployment risk, which is important for capturing the very largest earnings changes. On-the-job search is important because job-to-job transitions are associated with many of the largest earnings gains. We model multi-worker firms as this gives us a meaningful counterpart to revenue in the data and also allows firms to adjust by both laying off people and reducing earnings of some others within the same firm, which we could not speak to with a model of single worker firms. On the worker side, risk aversion is important to think about earnings risk. With risk neutral workers, it would not matter to individuals whether earnings fluctuate a lot or not. Finally, we want the model to have a reasonable wage setting mechanism to capture the fact that also some individuals who are continuously employed experience very large earnings changes. We model contracting under the assumption of two-sided limited commitment such that wages will be renegotiated infrequently.

Specifically, we develop an infinite horizon random search model in discrete time building on work by Lise and Robin (2017) and Gulyas (2020). Risk averse workers can be unemployed or employed and search on- and off-the-job. In addition to differences in their employment status they differ in their stochastic productivity. They receive unemployment benefits while unemployed and wages while employed. Risk-neutral firms differ in their stochastic productivity and firm size. Firms grow through costly vacancy posting and shrink if productivity shocks make some matches infeasible. Matches are created through random search, where meeting probabilities are governed by a standard matching function. Workers are paid a fixed wage, which can be renegotiated if one side has a credible threat to leave the match or the worker has an outside offer from another firm, as in Thomas and Worrall (1988) and Postel-Vinay and Robin (2002).

We calibrate the model to match key features of the Danish labor market such as labor market flows, wage inequality, the firm size distribution, and gross and net job creation and destruction. While the model calibration is still work in progress and the model does not yet match all targets well, it already does a very good job at endogenously reproducing key features of earnings risk, firm dynamics, and their interaction from the data. The worker earnings growth distribution is characterized by excess kurtosis such that many workers have stable earnings but some face very large changes. The same is true for the firm revenue growth distribution. The distributions of worker earnings growth in growing and shrinking firms have significant overlap with many individuals' earnings unaffected by their firms' performance. However, firms that experience negative productivity shocks reduce their workforce as some matches become infeasible and have a credible threat to renegotiate wages with some other workers. For these reasons the earnings growth distribution in shrinking firms has a significantly fatter left tail, as in the data.

With the calibrated model at hand, it is natural to ask which policies could be used to efficiently stabilize worker earnings and reduce the large welfare costs of idiosyncratic income risk. There are two categories of policies that could be used. First, policies can directly target worker earnings. Prominent examples that are used in most countries including Denmark are unemployment benefits and progressive income taxes, both of which are modeled. Second, given the strong relationship between firm growth and worker earnings growth, alternative policies could target firms in order to avoid or shorten unemployment spells. Such policies include layoff taxes or vacancy subsidies. We have run very preliminary experiments with progressive taxes and vacancy subsidies, which illustrate some key trade-offs. Progressive taxes have a very direct effect on reducing after-tax dispersion in earnings and earnings changes. However, there is a classic equity-efficiency trade-off with progressive taxes because high progressivity lowers vacancy posting and thereby increases unemployment in equilibrium. On the other hand, vacancy subsidies encourage vacancy posting, which lowers the unemployment rate in equilibrium. However, this is a costly policy, which has to be financed, thereby imposing other distortions. We plan to use the model to compare these and other policies rigorously in the future.

Related Literature. This paper is related to large literatures on earnings and firm dynamics on the empirical side and on firm and worker dynamics in frictional labor markets on the theoretical side.

Traditionally, earnings dynamics are modeled with innovations drawn from Gaussian distributions. This is the case for the literature estimating income processes following the seminal works of Lillard and Weiss (1979), MaCurdy (1981), and Abowd and Card (1989).[2] Normally distributed shocks are also the most common assumption for the income process used as input in incomplete markets models à la Bewley (1977), Huggett (1993), and Aiyagari (1994). More recent empirical evidence, however, strongly rejects the assumption of normality for income dynamics. Guvenen, Karahan, Ozkan, and Song (2021) document stark deviations from normality such as negative skewness and excess kurtosis of the earnings growth distribution applying non-parametric methods to administrative social security data from the U.S.[3] Arellano, Blundell, and Bonhomme (2017) provide evidence for nonlinear and nonnormal earnings dynamics using parametric methods applied to the PSID.

A number of papers investigate different dimensions of heterogeneity that are important for earnings risk. Guvenen, Ozkan, and Song (2014) study the cyclicality of earnings risk and argue that a key feature of cyclical earnings risk is countercyclical left-skewness of the earnings growth distribution (see also Storesletten, Telmer, and Yaron (2004), Pruitt and Turner (2020), and Busch, Domeij, Guvenen, and Madera (2021) for evidence on the cyclicality of earnings risk). Carrillo-Tudela, Visschers, and Wiczer (2021) investigate how employment and occupation changes affect earnings growth at different stages of the business cycle. Tanaka, Warren, and Wiczer (2020) link earnings growth to job flows. While most of the literature focuses on annual earnings, there are a few papers who decompose changes into hours and wages. Using French and Italian data, respectively, Pora and Wilner (2020) and Hoffmann and Malacrino (2019) find a dominant role of hours and unemployment in accounting for higher order moments of the earnings change distribution. Using Dutch data De Nardi, Fella, Knoef, Paz-Pardo, and

[2]For a summary of many studies in this literature, see the handbook chapter by Meghir and Pistaferri (2011).

[3]Earnings dynamics across a variety of countries are documented in the Global Income Dynamics Project organized by Guvenen, Pistaferri, and Violante. The project so far has produced evidence for Argentina (Blanco, Diaz de Astarloa, Drenik, Moser, and Trupkin, 2021), Brazil (Engbom, Gonzaga, Moser, and Olivieri, 2021), Canada (Bowlus, Gouin-Bonenfant, Liu, Lochner, and Park, 2021), Denmark (Leth-Petersen and Sæverud, 2021), France (Kramarz, Nimier-David, and Delemotte, 2021), Germany (Drechsel-Grau, Peichl, Schmieder, Schmid, Walz, and Wolter, 2021), Italy (Hoffmann, Malacrino, and Pistaferri, 2021), Mexico (Puggioni, Calderón, Zurita, Bujanda, González, and Jaume, 2021), Norway (Halvorsen, Ozkan, and Salgado, 2021), the United States (McKinney, Abowd, and Janicki, 2021), the United Kingdom (Bell, Bloom, and Blundell, 2021), Spain (Arellano, Bonhomme, De Vera, Hospido, and Wei, 2021), and Sweden (Friedrich, Laun, and Meghir, 2021).

Van Ooijen (2021) find some role for wages, but also argue in favor of a more important role for hours. Halvorsen, Holter, Ozkan, and Storesletten (2020) find important contributions of hours, wages, and their covariance to higher order moments of earnings dynamics in Norway. We also find that while unemployment spells are responsible for the very largest earnings drops, for continuously employed workers wage changes play an important role.

A separate empirical literature documents higher order moments for key economic outcomes on the firm side. Salgado, Guvenen, and Bloom (2019) provide evidence of procyclical skewness of sales growth for a variety of countries. Ilut, Kehrig, and Schneider (2018) document negative skewness in employment growth. Bachmann and Bayer (2013) and Kehrig (2015) focus on the cyclicality of productivity dispersion.

Finally on the empirical side there is a large literature linking worker wages to their firms. Card, Heining, and Kline (2013) and Song, Price, Guvenen, Bloom, and Von Wachter (2018) show that firm effects and sorting between workers and firms is important for understanding income inequality in levels using the methodology of Abowd, Kramarz, and Margolis (1999). Guiso, Pistaferri, and Schivardi (2005) estimate that there is significant pass-through from firm-level TFP shocks to wages. Their results have been replicated for a wide variety of countries.[4] Most closely related to this project Chan, Salgado, and Xu (2019) estimate heterogeneous pass-through from TFP to wages using Danish matched employer-employee data. In contrast to them, we also consider unemployment risk in addition to wages empirically and then move to a theoretical investigation using a random search model with large firms.

On the theoretical side, this paper is related to a large literature on firm and worker dynamics in frictional labor markets. Here, we highlight the relationship to two parts of this literature. First, this paper relates to a number of modeling contributions interested in endogenously producing the earnings process in search and matching models. Hubmer (2018) proposes a job ladder framework to replicate the evidence of Guvenen, Karahan, Ozkan, and Song (2021) on earnings risk over the life cycle. Lentz (2015), Tsuyuhara (2016), Ábrahám, Alvarez-Parra, and Forstner (2017), and Balke and Lamadon (2020) investigate optimal contracting between firms and workers in frictional labor markets. A number of papers proposes search and matching models to match the cyclical properties of the earnings growth distribution (Ai and Bhandari, 2021; Graber, 2018; Harmenberg and Sievertsen, 2017; McKay and Papp, 2012; Pascal, 2019).

Second, we contribute to a growing literature on search models with multi-worker firms. In the directed search tradition, important contributions are by Acemoglu and Hawkins (2014), Kaas and

[4]See for example Lagakos and Ordonez (2011) and Juhn, McCue, Monti, and Pierce (2018) for the U.S., Guertzgen (2014) for Germany, Fagereng, Guiso, and Pistaferri (2017) for Norway, and Friedrich, Laun, Meghir, and Pistaferri (2019) for Sweden.

Kircher (2015), and Schaal (2017). In the random search tradition, key papers include Elsby and Michaels (2013), Elsby and Gottfries (2019), and Bilal, Engbom, Mongey, and Violante (2019). Our framework builds in particular on the random search model of Gulyas (2020).

Roadmap. The paper proceeds as follows. In section 1.2 we provide empirical evidence using the Danish matched employer-employee data on firm and earnings dynamics and their interaction. In section 1.3 we introduce and quantify the model and take a preliminary look at policies. Section 1.4 concludes.

1.2 An Anatomy of Firm Dynamics and Earnings Risk

We start by providing evidence on worker earnings risk, firm dynamics, and their interaction using matched employer-employee data from Denmark.

1.2.1 Data

We use data from several Danish registries. Here, we describe only the most important features of the data. Details on the different data sources are relegated to Appendix A.1.

We focus on the time period from 2008 to 2018 because for that time period we have the highest quality information on employment spells and earnings. On the worker side, we observe labor earnings at a monthly frequency. We observe the start and end date of an employment spell within a month and an identifier for the firm at which the worker is employed. If a worker is employed at several firms within a single month we observe all these spells. The earnings measure is of high quality as it is third-party reported by tax authorities. Earnings are not top-coded. There is also an hours measure available, which allows us to construct a measure of hourly wages. The data set covers the entire population.

On the firm side, we observe the universe of firms. We can link the workers to the firms. In addition to the information on workers employed at the firms, we have information from accounting data on the firms. The key variable that we are going to focus on here to measure firm dynamics is firm revenue.

1.2.2 Evidence on Individual Earnings Risk

As a first step we provide evidence on the worker earnings growth distribution. This is similar to the evidence provided by Guvenen, Karahan, Ozkan, and Song (2021) for the United States. The measure of earnings that we use first is total annual earnings, in line with previous literature. To get this measure, we add up earnings from all monthly employment spells within a year that we observe in the data. We

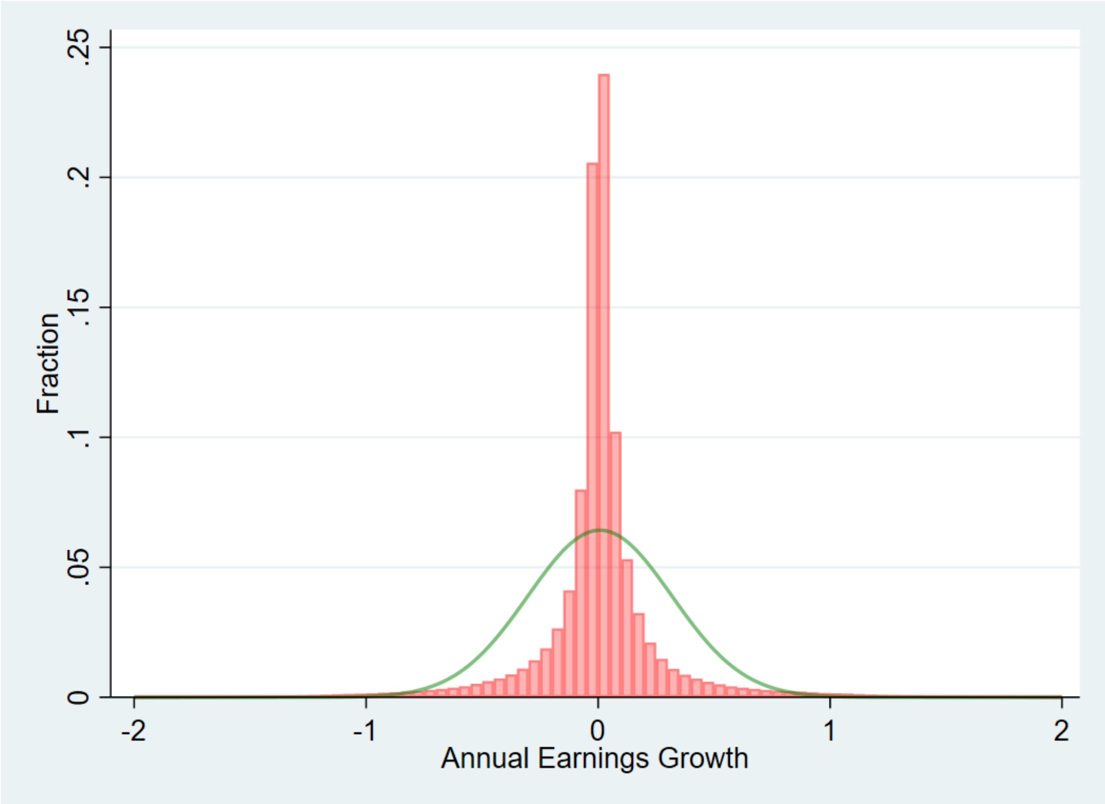


Figure 1.1 Annual Earnings Growth Distribution

Notes: This figure shows the annual earnings growth distribution for the entire sample. Growth rates are based on residual earnings taking out age, gender, and education effects. The green line compares a normal distribution with the same mean and standard deviation.

transform nominal to real earnings using the consumer price index. We impose a few sample restrictions that are standard in the earnings risk literature. We restrict ourselves to individuals who are between 25 and 60 years old. Minimum annual earnings are set to DKK20,000, which corresponds to roughly €2,700. We also impose that individuals work for at least 200 hours within a year. We compute earnings growth rates as log-differences.

The annual earnings growth distribution is shown in Figure 1.1. For this figure we use residualized earnings, where we take out age, gender, and education effects. However, in Appendix A.2 we show that the main features of the earnings growth distribution do not depend on this. Specifically, we replicate Figure 1.1 using raw labor earnings and also taking out occupation, industry, and location effects.

Figure 1.1 clearly shows that the earnings growth distribution is poorly approximated by a normal distribution, which is a common assumption in heterogeneous agent macroeconomics. A normal distribution implies that there are many people with medium sized earnings changes. In the data, however, there is much more mass around zero earnings changes than a normal distribution would

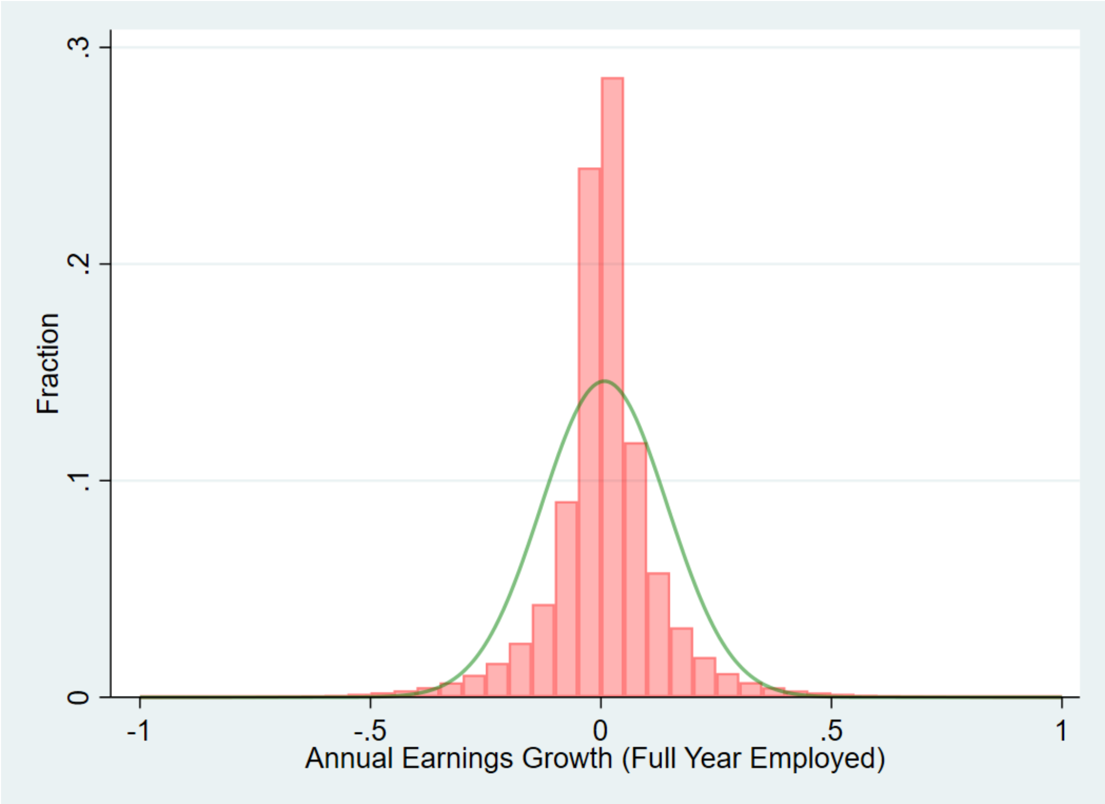


Figure 1.2 Annual Earnings Growth Distribution: Full-Year Employed

Notes: This figure shows the annual earnings growth distribution for individuals who are employed every month in two consecutive years. Growth rates are based on residual earnings taking out age, gender, and education effects. The green line compares a normal distribution with the same mean and standard deviation.

imply. On the other hand, there is also more mass in the extreme tails of the distribution. These features translate into excess kurtosis of the earnings growth distribution. The kurtosis of the annual earnings growth distribution is roughly 15, in contrast to 3 for a normal distribution. In slight contrast to the evidence of Guvenen, Karahan, Ozkan, and Song (2021) for the U.S. the distribution is almost symmetric. The coefficient of skewness is only very slightly negative with a value of -0.03.[5]

The key feature of the earnings growth distribution thus is that individuals experience very small earnings changes most of the time, but face a significant chance of a very large change to their earnings. This result could in principle be only driven by transitions to and out of unemployment. Individuals who lose their jobs see their earnings drop drastically and see a large rise when they are employed again. To investigate whether the deviations from normality are only driven by unemployment, we next restrict

[5]The skewness of the earnings growth distribution exhibits the cyclical properties that have been documented in other countries. The skewness is lower, and significantly negative, in the recession year 2009 and higher in the expansion years later in the sample. We show the earnings growth distributions for the years 2009 and 2015 in Appendix A.2. There are many more individuals with very large earnings drops in recessions, whereas the likelihood of earnings being stable or going up significantly is higher in expansions.

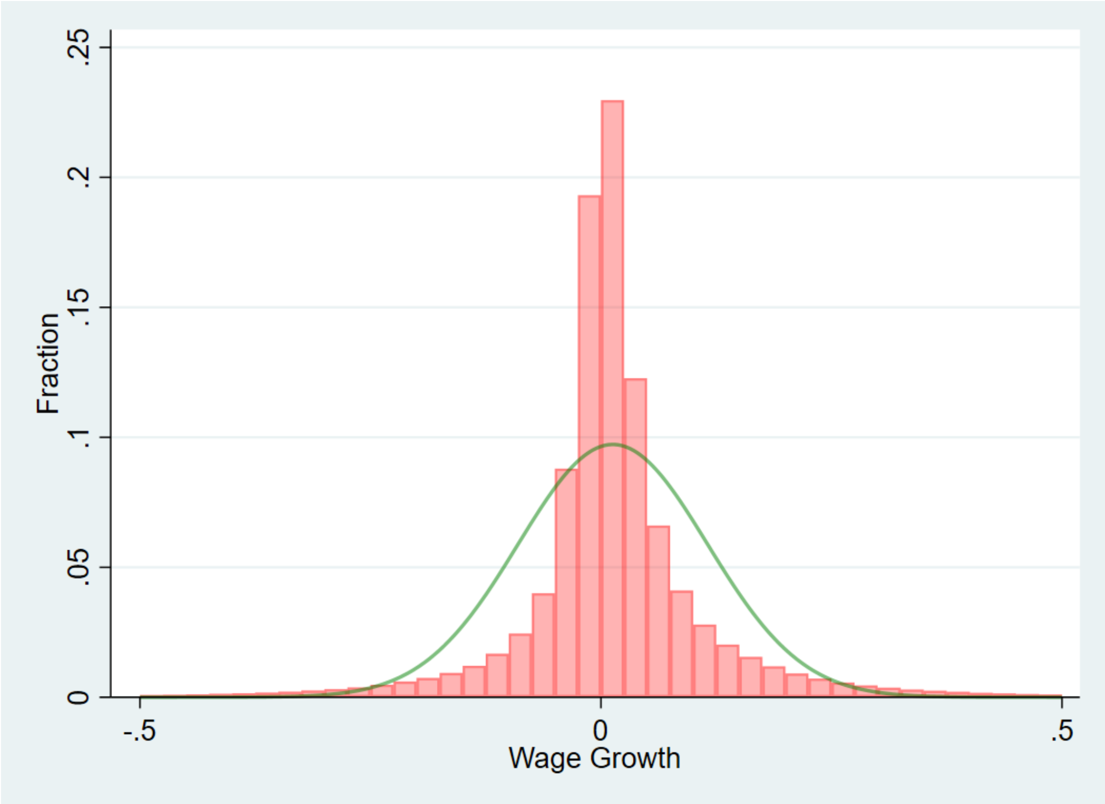


Figure 1.3 Annual Wage Growth Distribution: Full-Year Employed

Notes: This figure shows the annual wage growth distribution for individuals who are employed every month in two consecutive years. Growth rates are based on residual wages taking out age, gender, and education effects. The green line compares a normal distribution with the same mean and standard deviation.

our sample to individuals who are employed in every month of two consecutive years. The earnings growth distribution for this group is shown in Figure 1.2. The standard deviation of earnings growth for this subsample is significantly lower than for the entire population, 0.15 instead of 0.34, indicating that unemployment risk is a key part of earnings risk. However, the earnings growth distribution of the full-year employed exhibits otherwise similar features compared to the earnings growth distribution of the entire population. Relative to a normal distribution with the same mean and standard deviation it is much more likely to have very small or very large earnings changes.

Finally on the worker side, we investigate the distribution of growth rates of hourly wages. The definition of the hourly wage is annual earnings divided by annual hours. Again, the standard deviation of the hourly wage growth distribution is significantly lower than that of the annual earnings growth distribution because unemployment spells play an important role in driving up earnings variability. However, the hourly wage growth distribution shares with the annual earnings growth distribution excess kurtosis as a key feature.

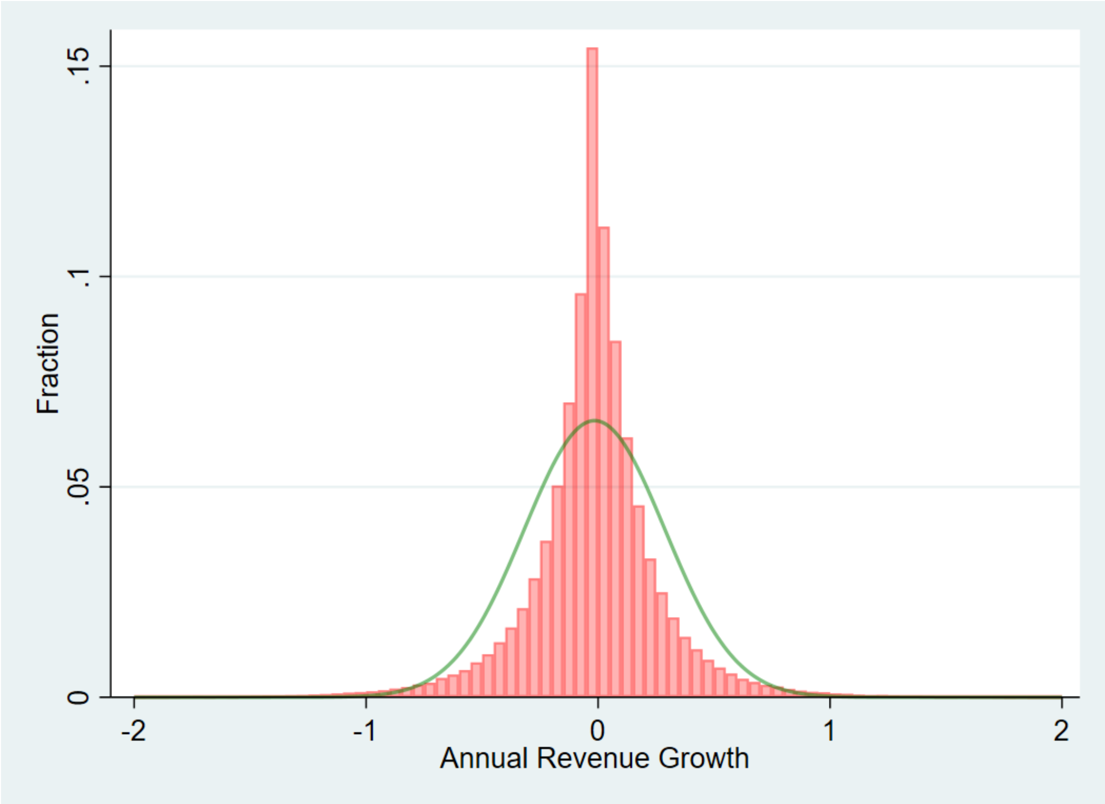


Figure 1.4 Annual Revenue Growth Distribution

Notes: This figure shows the annual revenue growth distribution. The green line compares a normal distribution with the same mean and standard deviation.

1.2.3 Evidence on Firm Dynamics

Before investigating the relationship between firms and the worker earnings growth distribution we provide evidence on the key variable that we are going to use on the firm side, which is firm revenue. For our main analysis, we are going to focus on revenue that we obtain from accounting/tax information. This is not available for all firms; in particular, while this information is available for the majority of large firms, it is available only for a small subset of small firms. However, the firms for which the information is available account for the majority of employment. In Appendix A.2 we provide additional evidence where we compute revenues from value added tax forms. This is available for many more firms. None of our main conclusions are affected by the ex- or inclusion of these firms.

In Figure 1.4 we show the distribution of annual firm revenue growth.[6] The distribution shares many features with the distribution of worker earnings growth. There is a lot of mass around zero revenue changes, much more than implied by a normal distribution. Also, the distribution is fat tailed.

[6]In the appendix we also provide evidence on the value added growth distribution.

Excess kurtosis is even higher than for the worker earnings growth distribution. There is a significant mass of firms whose revenues shrink and grow drastically, while most firms have very stable revenues. This raises the question about the relationship between firm growth and the worker earnings growth distribution. Are all the worker who experience large earnings drops employed in shrinking firms? Or is the earnings growth distribution largely decoupled from firm outcomes as firms buffer workers from large earnings fluctuations due to firm-level revenue changes? We now turn to addressing these questions.

1.2.4 Evidence from Matched Firm-Worker Data

To analyze the comovement of worker earnings and firm revenues we match workers to firms in year $t-1$. Then, we group firms by their revenue growth from $t-1$ to t. We compute the worker earnings growth distribution from $t-1$ to t by firm group. In $t-1$ we impose the following sample restrictions: Workers have to be employed for the entire year. Furthermore, they may only be employed at one firm during the entire year. Then, we consider four cases. First, we compute the earnings growth distribution not conditioning on anything in year t. That is, a worker may still be employed at the same firm as in $t-1$, she may be employed at a different firm, she may be unemployed, or any combination of these. Second, we compute the earnings growth distribution by firm group, conditional on also being employed in the same firm for the entire year t. Then, we want to decompose earnings changes into changes to wages and changes to hours. Hence, third we consider the wage growth distribution for those who are employed in two consecutive years at the same firm by firm growth group. Fourth, we compute the hours growth distribution.

In Figure 1.5 we show the earnings growth distribution for two firm groups, a firm group whose revenues shrink by 25 to 30% and a firm group whose revenues grow by 25 to 30%. Note that because of the restriction that workers have to be full-year employed at the firm in year $t-1$ but can be unemployed in year t average earnings growth tends to be negative. Mean earnings growth is, however, significantly different across firm groups. In the firms whose revenues shrink by 25 to 30%, mean annual earnings growth of workers is around -7%. By contrast, in the growing firms mean earnings growth is roughly zero. However, there is significant overlap in the earnings growth distributions. Even in the firms that shrink quite strongly there are many workers whose earnings are either barely affected or even grow. However, large earnings growth is much more likely in the growing firms, whereas the likelihood of large earnings drops is much higher in shrinking firms

We provide a different visualization taking into account all firm groups in Figure 1.6. In the upper left panel we plot the coefficients from an OLS regression of earnings growth on firm group and year

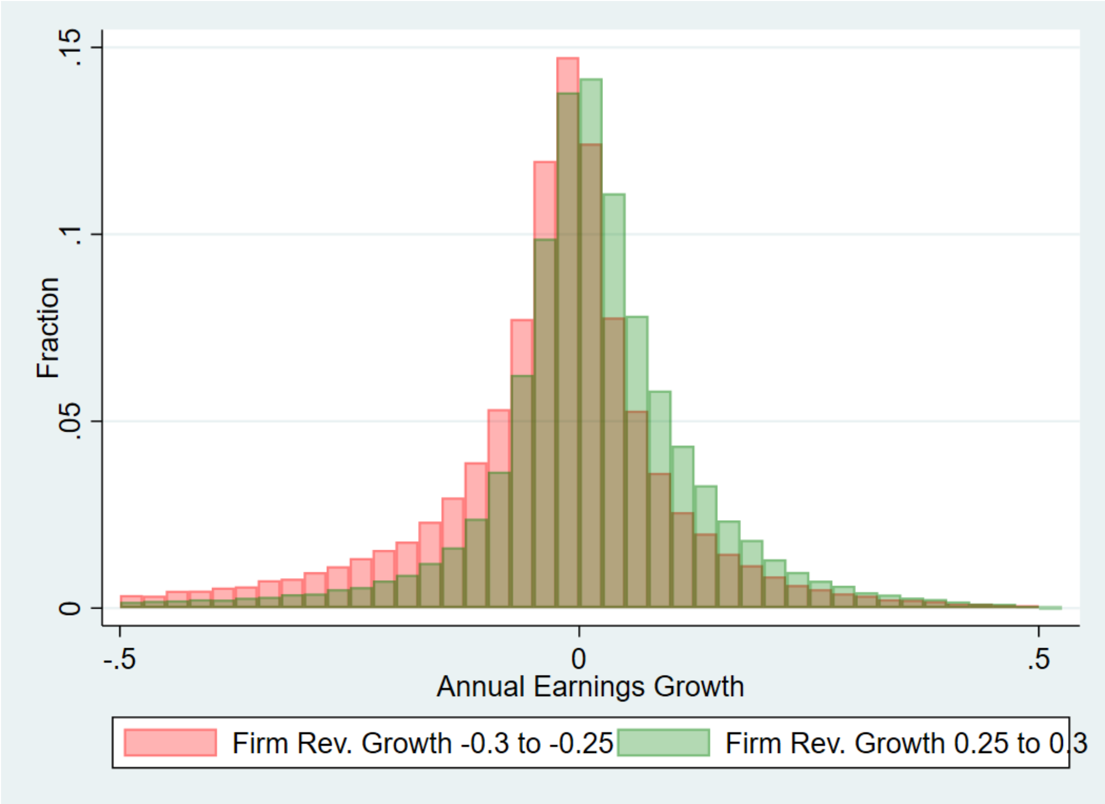


Figure 1.5 Worker Earnings Growth by Firm Revenue Growth

Notes: This figure shows the annual worker earnings growth distribution conditional on firm revenue growth.

dummies. The omitted firm group is the one with the lowest growth rate. Hence, we can see that average earnings growth is monotonically increasing in firm revenue growth. The average difference between the lowest and the highest firm revenue growth group is almost ten percentage points. In the other panels of the figure we repeat the regression exercise, but instead of running an OLS regression we use quantile regressions (upper right panel: 10th percentile; lower left panel: 50th percentile; lower right panel: 90th percentile).

This figure also clearly shows that what is most affected by firm growth is the bottom of the earnings growth distribution. The difference in the 10th percentile of the earnings growth distribution between the lowest and the highest firm group is almost twenty percentage points. The median and the 90th percentile of the earnings growth distribution are much less affected.

As already shown for the distribution of worker earnings growth on its own, unemployment risk is an important driver of the tails of the earnings growth distribution. The likelihood of having an unemployment spell is also closely associated with the growth of the firm at which an individual is employed. Out of the workers who are employed in the lowest firm group roughly 14% spend at least

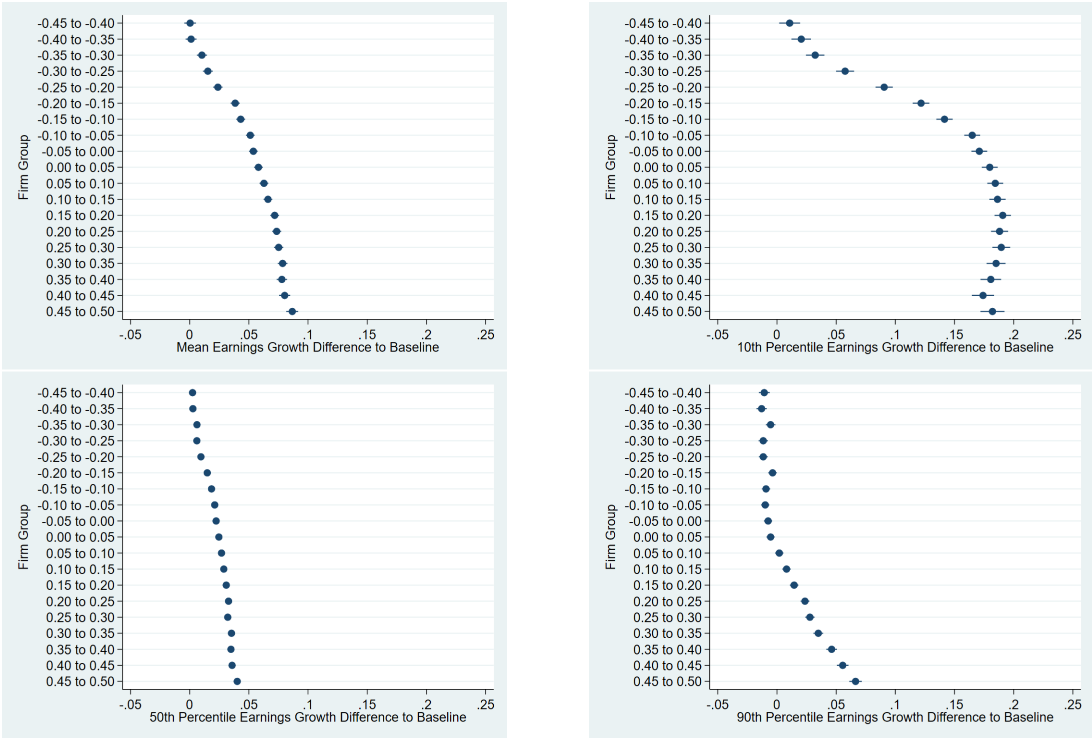


Figure 1.6 Worker Earnings Growth by Firm Revenue Growth

Notes: This figure shows the the relationship between firm performance and different parts of the earnings growth distribution.

one month unemployed in the next year. In the median firm groups this probability is only 6.5%. It does not change much, however, from the median firm groups to the very top firm groups.

Thus, firm growth matters for the earnings growth distribution through its association with the probability of becoming unemployed. In Figure 1.7 we show that firm growth is also related to the earnings growth distribution of the workers who are continuously employed at the same firm for two full consecutive years. For that purpose, we impose the additional restriction that a worker is employed also in year t at the same firm for the entire year and at no other firm.

Because we are now looking at a sample of continuously employed individuals average earnings growth is not negative by construction anymore. A sizeable difference between average earnings growth depending on firm growth, however, remains. Mean earnings growth of workers employed in firms with a revenue drop of 25 to 30% is -1.3%, while it is +2.0% for workers employed in firms with a revenue increase of 25 to 30%. Again, this difference in means is mostly driven by a much larger chance of large earnings rises in growing firms and a much larger probability of large earnings losses in shrinking firms, whereas there is a lot of overlap in the distribution with many workers in both firm groups experiencing

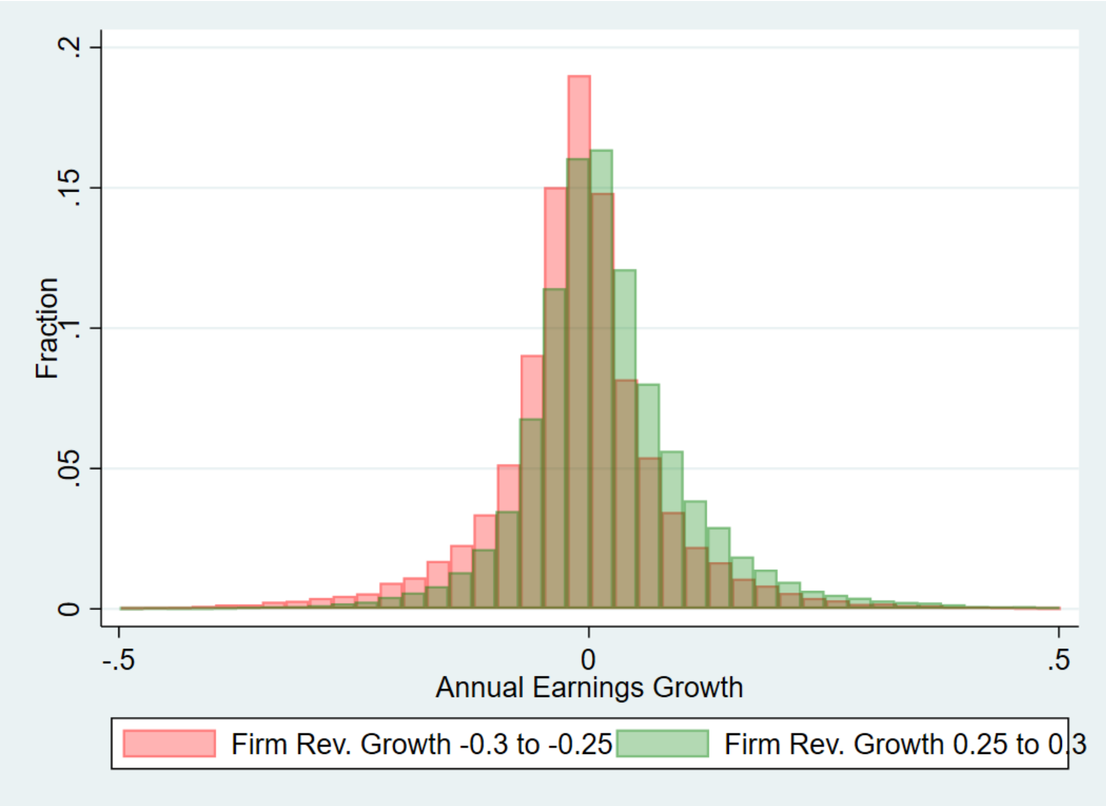


Figure 1.7 Worker Earnings Growth by Firm Revenue Growth: Stayers

Notes: This figure shows the annual worker earnings growth distribution conditional on firm revenue growth for workers who are continuously employed at the same firm for two years.

very small earnings changes. We relegate the corresponding figure with regression coefficients for all firm groups to the appendix.

Finally, we investigate whether these earnings changes of individuals who are continuously employed are mostly due to wages or hours. Figure 1.8 shows the wage growth distribution, where wages are annual earnings divided by annual hours, for the two firm groups, conditioning on workers being continuously employed at the same firm. The picture that emerges is quite similar to the earnings growth distribution: Mean wage changes are -0.7% and 2.2% in the two groups. While there is again considerable overlap among the two distributions, the distribution in the growing firms has a much larger right tail, whereas the distribution in the shrinking firms has a much larger left tail. Again, the figure for all firm groups is in the appendix.

By contrast, there is no strong relationship between hours growth of workers and firm growth. This is shown in Figure 1.9. While there is a weak positive relationship between hours growth of workers and firm growth, this relationship is not strong enough to explain the strong association between firm growth and worker earnings growth. In the main text we restrict ourselves to workers for whom hours

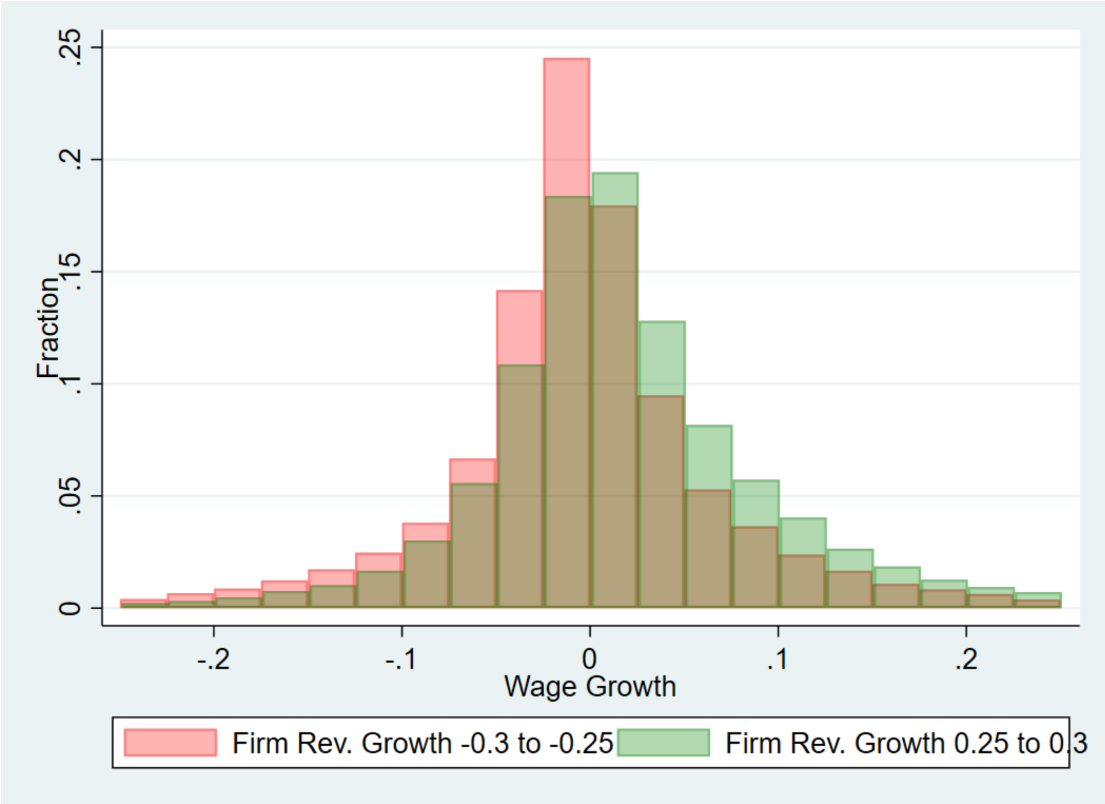


Figure 1.8 Worker Wage Growth by Firm Revenue Growth: Stayers

Notes: This figure shows the worker wage growth distribution conditional on firm revenue growth for workers who are continuously employed at the same firm for two years.

worked are reported. In the appendix, we replicate the figure using an hours measure that is imputed by Statistics Denmark, so that more workers are part of the sample. While hours growth based on this measure is more dispersed overall, there is still no strong enough relationship between firm growth and hours growth to account for the sizeable association between firm growth and annual earnings growth.

1.2.5 Taking Stock

In the empirical part we have documented that the individual earnings process exhibits stark deviations from normality. Most individuals have very stable earnings from year to year, but every once in a while there are very large changes. This is the case for the annual earnings growth distribution (including unemployment spells), but also holds (with an overall lower standard deviation) for the annual earnings growth and wage growth distributions of the continuously full-year employed. We also show that there is a strong relationship between firm growth and the worker earnings growth distribution. There is a much longer left tail of the earnings growth distribution in shrinking firms. This is partially driven by separations, but also by wages.

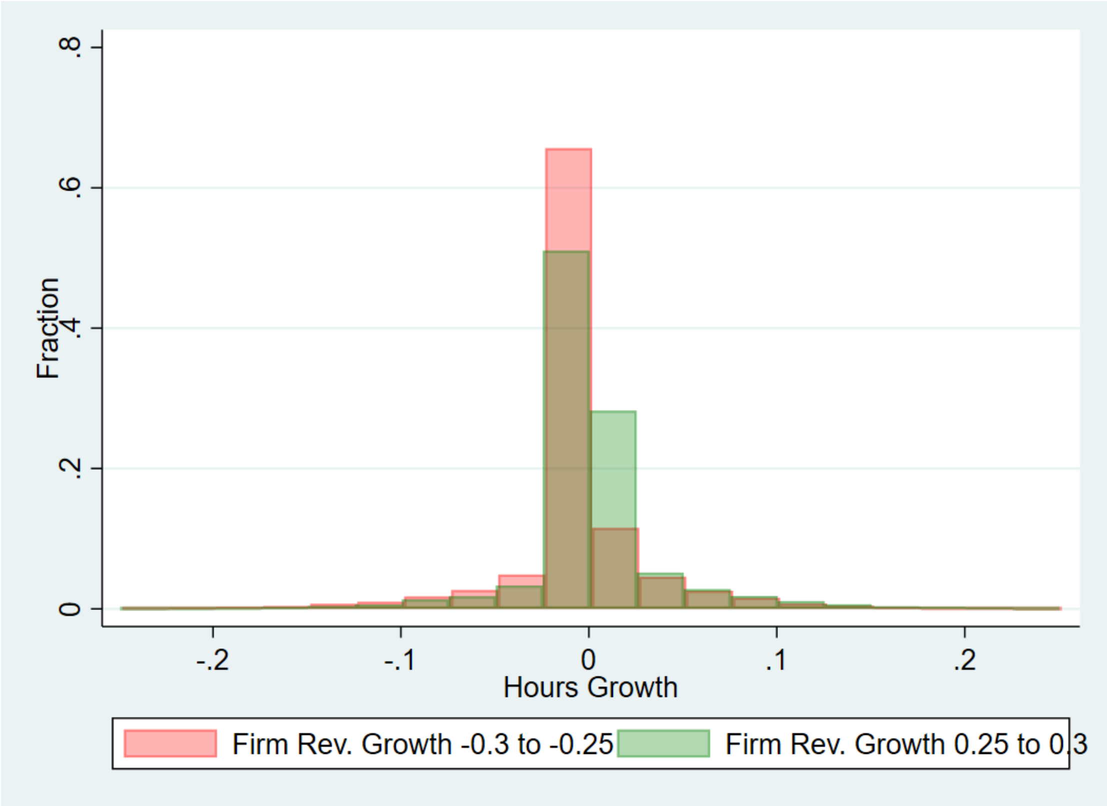


Figure 1.9 Worker Hours Growth by Firm Revenue Growth: Stayers

Notes: This figure shows the hours growth distribution conditional on firm revenue growth for workers who are continuously employed at the same firm for two years.

These patterns are not unique to Denmark. In Appendix A.3, we include similar evidence on the basis of German matched employer-employee data. While we cannot compute everything we compute for the Danish case with the German data because there is no information on hours/wages, we confirm with this data the relationship between worker earnings growth and firm revenue growth that we see in the Danish data, both when including unemployment spells and when restricting attention to job stayers. Motivated by this evidence, we now turn to a model of firm dynamics and earnings risk.

1.3 A Model of Firm Dynamics and Earnings Risk

1.3.1 Model Setup

The model is a random search model with multi-worker firms and heterogeneous workers in discrete time. It builds on Lise and Robin (2017) and in particular on Gulyas (2020).

Workers. The economy is populated by a unit mass of infinitely lived workers, who are heterogeneous in their labor productivity x. Worker productivity is stochastic with stationary distribution $\phi_x(x)$. It follows a first order Markov process with conditional transition probabilities denoted by $p(x' \mid x)$. Workers can be either employed or unemployed. If unemployed, they receive an unemployment benefit $b(x)$ and search for jobs. If employed, they receive a wage w and search on the job with exogenously lower search intensity s. Arrival rates of job offers are determined in equilibrium.

Workers are assumed to be hand-to-mouth; they cannot save or borrow. Hence, consumption is equal to the wage or benefit they receive every period. Households value consumption according to a utility function $u(c)$, which is assumed to be increasing and concave in c such that workers are risk averse. Workers' discount factor is β.

Firms. There is a mass M of firms, who can employ many workers. Firm productivity y is also stochastic with stationary distribution $\phi_y(y)$. Firm productivity is also first order Markov with transition probabilities $p(y' \mid y)$. Firms create jobs, which can be either vacant or filled. Job creation is a costly process: The job creation cost function $c(v_j)$, where v_j denotes the measure of newly created jobs of firm j, is increasing and convex in v_j. Jobs are costless to maintain, but are destroyed exogenously with probability δ. A job filled with a worker of type x produces output $f(x,y)$. This output is independent of other matches, so that output of firm j is given by the integral of output over all its matches

$$F_j(y) = \int f(x,y)\, d\psi_j(x). \tag{1.1}$$

Vacant jobs meet a searching worker with a probability that is determined in equilibrium. Firms maximize expected profits and are risk-neutral. They also discount future profits with discount factor β.

Government. The government pays unemployment benefits to unemployed workers. These benefits potentially depend on worker type and are denoted $b(x)$. Furthermore, the government raises income taxes using a progressive income tax function $T(w)$. Additionally, the government has access to a vacancy subsidy τ_V, which proportionally reduces the cost of creating a vacancy, and a layoff tax τ_L, which has to be paid if there is an endogenous separation. These latter two instruments are zero in the calibration, but will be explored in the policy exercises.

Search and Matching. In a steady state equilibrium, there are distributions of unemployed $\mu_x(x)$, of vacant jobs $\mu_y(y)$, and of matches at the search stage $\psi^s(x,y)$. We denote the total measure of unemployed with u, the measure of employed at the search stage as e^s, and the measure of vacant jobs as v. Because on the job search is assumed to have exogenously lower search intensity s, effective search units are given by

$$L = u + se^s. \tag{1.2}$$

The number of meetings is given by a standard matching function

$$M = \min\{M(L,v), L, v\}. \tag{1.3}$$

Then, the probability of a vacant job meeting a worker λ_f, the probability of an unemployed worker meeting a vacant job λ_u, and the probability of an employed worker meeting a vacant job λ_e are given by

$$\lambda_f = \frac{M}{v},$$
$$\lambda_u = \frac{M}{L},$$
$$\lambda_e = s\lambda_u.$$

Also, denote with p_u the probability that a worker met by a vacant job is unemployed, conditional on a meeting happening:

$$p_u = \frac{\lambda_u u}{\lambda_u u + \lambda_e e^s}. \tag{1.4}$$

If a vacant job and an unemployed worker meet, given firm and worker productivity there is a maximum wage the firm is willing to pay, $\bar{w}_y(x,y)$, and a minimum wage the worker is willing to accept, $\underline{w}_x(x,y)$. As long as the firm can pay more than the worker requires, the match is created. If a vacant job meets an employed worker, there will be a job-to-job transition if the potential employer can pay a higher wage than the incumbent. Otherwise, the worker stays at the incumbent firm.

Wage Setting. If an unemployed worker meets a vacant job and it is possible to create a match, the assumption is that the worker has some bargaining power to extract a share of the surplus. Specifically, we assume that the initial wage negotiated between a firm of type y and a worker of type x is an average between the maximum wage the firm would pay and the minimum the worker would accept:

$$w^{\text{init}}(x,y) = (1-\alpha)\,\underline{w}_x(x,y) + \alpha\bar{w}_y(x,y). \tag{1.5}$$

We will explain how to compute $\underline{w}_x(x,y)$ and $\bar{w}_y(x,y)$ below, after having introduced the value functions.

Employed workers can renegotiate their wages if they receive a relevant outside offer. Suppose a worker employed at a firm with productivity y, which is able to pay at most $\bar{w}_y(x,y)$, meets another firm with productivity $\tilde{y}$ being able to pay $\bar{w}_y(x,\tilde{y})$. Three cases can occur. First, if the maximum the potential poaching firm can pay is below the current wage, this is an irrelevant outside offer and nothing will happen. Second, the current firm is able to pay more than the potential new firm, but the current wage is lower than the wage this firm could pay. Then, the wage is increased to this level, but the worker stays at the old firm. Third, if the new firm can pay a higher wage than the old firm, there will be a job-to-job transition and the worker will receive the maximum wage the old firm could have paid as the starting wage at the new firm. If the two firms have the same productivity, the tie breaking rule we assume is that there is a job-to-job transition in 50% of the cases.

Even without an outside offer wages may change while a worker is employed. We assume that either party can demand a renegotiation of the wage if it has a credible threat to leave the match. For the worker that means that the wage can be renegotiated if quitting to unemployment is preferable to staying in the match at the current wage. For the firm it means that it can ask for a renegotiation if expected profits are lower than the layoff tax. In both cases, wages are reset such that the agent demanding the renegotiation is just indifferent between staying in the match or leaving.

Timing. The order of events in a period is as follows. At the beginning of the period there is the production stage. The distribution of matches at the production stage is denoted with $\psi(x,y)$. Output is produced, wages and benefits are paid out, and consumption takes place. Furthermore, firms decide how many new vacant jobs to create.

After production and the creation of new vacant jobs, exogenous job destruction shocks occur. This implies that a newly created vacant job can be destroyed immediately. If a filled job is destroyed, the worker becomes unemployed. A newly separated worker cannot search immediately but only in the next period, so that a separated worker will be unemployed for at least one period.

Next, productivity shocks realize. This can lead to endogenous separations. If a match is dissolved, the job becomes vacant and the worker transitions to unemployment. Again, unemployed workers have to be unemployed for at least one period and the job also has to be vacant for a period.

A period concludes with the matching stage. Meetings realize, new matches between unemployed searchers and vacant jobs are created, and job-to-job transitions take place. The relevant distributions at this stage are the distribution of matches at the search stage, $\psi^S(x,y)$, the distribution of unemployed workers $\mu_x(x)$, and the distribution of vacant jobs $\mu_y(y)$.

Value Functions. We can now write the value functions of an unemployed worker, an employed worker, a vacant job, and a filled job. We start with the value function of an unemployed worker:

$$
\begin{aligned}
U(x) = {} & u(b(x)) \\
& + \beta(1-\lambda_u)\int_{x'} U(x')\, p(x' \mid x)\, dx' \\
& + \beta\lambda_u \int_{x'}\int_{\tilde{y}} \left(1 - A_U(x',\tilde{y})\right) U(x') \frac{\mu_y(\tilde{y})}{v} p(x' \mid x)\, d\tilde{y}dx' \\
& + \beta\lambda_u \int_{x'}\int_{\tilde{y}} A_U(x',\tilde{y})\, W\left(x',\tilde{y},w^{\text{init}}(x',\tilde{y})\right) \frac{\mu_y(\tilde{y})}{v} p(x' \mid x)\, d\tilde{y}dx'.
\end{aligned}
\tag{1.6}
$$

The value of unemployment consists of four components. An unemployed worker obtains instantaneous utility from consuming the unemployment benefit, which can be read off the first line of Equation (1.6). The next three lines of Equation (1.6) sum up to the continuation value. There are three relevant scenarios to consider for an unemployed worker. First, the worker may not meet a vacant job, so that she will remain unemployed. In that case, she will continue to receive the value of unemployment tomorrow, taking into account potential productivity shocks (line 2). Second, the worker may meet a new firm, but productivities are such that no match is created. Let $A_U(x,y)$ denote the probability that a match between a worker of type x and a firm with productivity y is created. This probability is one if the firm is able to pay a higher wage than the minimum wage the worker requires to enter the match and zero otherwise. For whether a match is created or not, the worker has to take into account her potential productivity states one period ahead and the probabilities of meeting a vacant job of a certain type from the distribution of vacant jobs μ_y. If no match is created, the worker will continue to receive the value of unemployment (line 3). Line 4 covers the last possibility: The worker meets a vacant job and a match is created, in which case the worker's continuation utility is $W\left(x',\tilde{y},w^{\text{init}}(x',\tilde{y})\right)$, the value of employment given worker productivity, firm productivity, and the wage.

The value of employment can be written as follows:

$$
\begin{aligned}
W(x,y,w) = {} & u(w - T(w)) \\
& + \beta\delta \int_{x'} U(x') \, p(x' \mid x) \, dx' + \beta(1-\delta) \Bigg\{ \\
& + \int_{x'} \int_{y'} (1 - A_S(x',y')) \, U(x') \, p(x' \mid x) \, p(y' \mid y) \, dx' dy' \\
& + \int_{x'} \int_{y'} A_S(x',y') \int_{\tilde{y}} \lambda_E A_E(x',y',\tilde{y}) \, W(x',\tilde{y},\bar{w}_y(x',y')) \, \frac{\mu_y(\tilde{y})}{v} p(x' \mid x) \, p(y' \mid y) \, d\tilde{y} dx' dy' \\
& + \int_{x'} \int_{y'} A_S(x',y') \int_{\tilde{y}} \lambda_E A_{OO}(x',y',w,\tilde{y}) \, W(x',y',\bar{w}_y(x',\tilde{y})) \, \frac{\mu_y(\tilde{y})}{v} p(x' \mid x) \, p(y' \mid y) \, d\tilde{y} dx' dy' \\
& + \int_{x'} \int_{y'} A_S(x',y') \int_{\tilde{y}} (1 - \lambda_E \left[A_E(x',y',\tilde{y}) + A_{OO}(x',y',w,\tilde{y})\right]) \\
& \times \min\left\{\max\left\{W(x',y',w), U(x')\right\}, W(x',y',\bar{w}_y(x',y'))\right\} \frac{\mu_y(\tilde{y})}{v} p(x' \mid x) \, p(y' \mid y) \, d\tilde{y} dx' dy' \Bigg\}.
\end{aligned}
\tag{1.7}
$$

Again, the first line of Equation (1.7) describes the instantaneous utility. The worker consumes the after tax wage. With probability δ, the worker is exogenously separated from the job and transitions to unemployment, which is covered in line 2. Furthermore, note the probability $1-\delta$, which is the probability of not being exogenously separated, in line 2, which applies to all the following cases. If the worker is not exogenously separated, four more cases have to be considered. Line 3 takes care of the case in which there is an endogenous separation. $A_S(x,y)$ denotes the probability of a match continuing to exist, so that $1 - A_S(x,y)$ is the probability of an endogenous separation. Note that absent a layoff tax $A_S(x,y) = A_U(x,y)$. Line 4 deals with job-to-job transitions. A job-to-job transition requires that an employed worker is not endogenously separated before new meetings take place and that a meeting with a vacant job occurs, which is the case with probability λ_E. Then, $A_E(x,y,\tilde{y})$ denotes the probability that a worker transitions to the new job given worker productivity x, incumbent firm productivity y, and potential employer productivity $\tilde{y}$. This probability is one if the potential poacher can pay a higher wage than the incumbent and zero otherwise. The worker receives the maximum wage the incumbent employer could have paid, $\bar{w}_y(x',y')$. An outside offer that does not lead to a job-to-job transition may still trigger a wage renegotiation, which is considered in line 5. $A_{OO}(x,y,w,\tilde{y})$ is the probability of a wage renegotiation between a worker of productivity x employed at a firm of productivity y with wage w, who meets a vacant job with productivity $\tilde{y}$. Then, the wage the worker receives is the maximum the potential new employer would have been willing to pay, $\bar{w}_y(x',\tilde{y})$. The continuation utility from the scenario in which the worker is neither exogenously nor endogenously separated and does not receive a relevant outside offer triggering either a job-to-job transition or a renegotiation is covered in the last two lines. If the worker's value of employment falls below the outside option, which is the value of unemployment, the wage is reset such that the worker receives the value of unemployment. If, by contrast, the wage is higher than what the firm can maximally pay after productivity shocks, the wage will be lowered to this maximum. If neither of these things happen, the wage will remain the same.

The value functions for a vacant job and for a filled job follow a similar logic. Consider first the value of a vacant job:

$$
\begin{aligned}
V(y) = \beta(1-\delta)\Bigg\{ & (1-\lambda_f)\int_{y'} V(y')\, p(y' \mid y)\, dy' \\
& + \lambda_f p_u \int_{y'}\int_{\tilde{x}} \left(1 - A_U(\tilde{x}, y')\right) \frac{\mu_x(\tilde{x})}{u} V(y')\, p(y' \mid y)\, d\tilde{x} dy' \\
& + \lambda_f p_u \int_{y'}\int_{\tilde{x}} A_U(\tilde{x}, y') \frac{\mu_x(\tilde{x})}{u} J\left(\tilde{x}, y', w^{\text{init}}(\tilde{x}, y')\right) p(y' \mid y)\, d\tilde{x} dy' \\
& + \lambda_f (1-p_u) \int_{y'}\int_{\tilde{x}}\int_{\tilde{y}} \left(1 - A_E(\tilde{x}, \tilde{y}, y')\right) \frac{\psi^S(\tilde{x}, \tilde{y})}{e^S} V(y')\, p(y' \mid y)\, d\tilde{y} d\tilde{x} dy' \\
& + \lambda_f (1-p_u) \int_{y'}\int_{\tilde{x}}\int_{\tilde{y}} A_E(\tilde{x}, \tilde{y}, y') \frac{\psi^S(\tilde{x}, \tilde{y})}{e^S} J\left(\tilde{x}, y', \bar{w}_y(\tilde{x}, \tilde{y})\right) p(y' \mid y)\, d\tilde{y} d\tilde{x} dy' \Bigg\}.
\end{aligned}
\tag{1.8}
$$

A vacant job delivers no instantaneous value and is costless to maintain. The continuation values are discounted with discount factor β and a vacant job is destroyed with probability δ. Only if the job is not exogenously destroyed is there a continuation value. Line 1 of Equation (1.8) gives the continuation value for the case in which the vacant job does not meet a worker. In that case, the vacant job continues to exist, but is subject to firm-level productivity shocks. The remaining four cases all deal with instances in which the vacant job meets a worker. In the first two of these cases, the worker is unemployed. Line 2 gives the continuation value for the productivities in which a match is not created. Line 3 provides the continuation value for the case in which the job is filled with the unemployed worker. The last two cases account for meetings with employed workers. Line 4 takes care of the cases in which the worker does not do a job-to-job transition towards the firm characterized by future productivity y', so that the continuation value remains the value of a vacant job. Line 5 finally accounts for filling the vacant job through poaching a worker from a different firm.

Finally, consider the value of a filled job:

$$
\begin{aligned}
J(x,y,w) &= f(x,y) - w + \beta(1-\delta)\Bigg\{ \\
&+ \int_{x'}\int_{y'} (1 - A_S(x',y'))\left[V(y') - \tau_L\right] p(x' \mid x)\, p(y' \mid y)\, dx'dy' \\
&+ \int_{x'}\int_{y'} A_S(x',y') \int_{\tilde{y}} \lambda_E A_E(x',y',\tilde{y})\, V(y') \frac{\mu_y(\tilde{y})}{v} p(x' \mid x)\, p(y' \mid y)\, d\tilde{y}dx'dy' \\
&+ \int_{x'}\int_{y'} A_S(x',y') \int_{\tilde{y}} \lambda_E A_{OO}(x',y',w,\tilde{y})\, J(x',y',\bar{w}_y(x',\tilde{y})) \frac{\mu_y(\tilde{y})}{v} p(x' \mid x)\, p(y' \mid y)\, d\tilde{y}dx'dy' \\
&+ \int_{x'}\int_{y'} A_S(x',y') \int_{\tilde{y}} \left(1 - \lambda_E\left[A_E(x',y',\tilde{y}) + A_{OO}(x',y',w,\tilde{y})\right]\right) \\
&\times \min\left\{\max\left\{J(x',y',w), V(y') - \tau_L\right\}, J(x',y',\underline{w}_x(x',y'))\right\} \frac{\mu_y(\tilde{y})}{v} p(x' \mid x)\, p(y' \mid y)\, d\tilde{y}dx'dy' \Bigg\}.
\end{aligned}
\tag{1.9}
$$

The instantaneous value of a filled job, given in line 1 of Equation (1.9), is the output of the match between the firm with productivity y and the worker with productivity x minus the wage paid to the worker. Note that if the job is exogenously destroyed there is no continuation value for the firm in contrast to the worker, who transitions to unemployment. Line 2 deals with endogenous separations: If productivity shocks are such that an endogenous separation is optimal, the firm will have a vacant job and has to pay the layoff tax. Lines 3 and 4 account for the cases in which the worker meets another vacant job. Such an outside offer may result in either a job-to-job transition, so that the firm is left with a vacant job (line 3), or in a renegotiation, so that the firm keeps the worker but has to increase the wage (line 4). Finally, lines 5 and 6 take care of the case in which there is no exogenous separation, endogenous separation, or relevant outside offer. If the firm prefers a separation at the current wage, the wage will be reset such that the firm is indifferent between an endogenous separation and continuing the match. If the worker has a credible threat to leave the match at the current wage, the wage will be increased to make the worker indifferent between leaving and staying. If neither agent has a credible threat, the match will continue at the current wage.

The value functions can be used to compute the maximum wage a firm is willing to pay and the minimum wage a worker is willing to accept. The lowest wage a worker of type x accepts at a firm of type y is defined as follows:

$$
\underline{w}_x(x,y):\; W(x,y,\underline{w}_x(x,y)) = U(x). \tag{1.10}
$$

The highest wage a firm of type y is willing to pay to a worker of type x is given by

$$\bar{w}_y(x,y): \; J(x,y,\bar{w}_y(x,y)) = V(y). \tag{1.11}$$

In a model with risk neutral workers such as Gulyas (2020) it would be possible to simplify the problem drastically and solve for allocations only using the surplus function, which would only be a function of productivities but not of wages. Because of risk neutrality the wage is just a transfer between firm and worker without any effect on the surplus. The assumptions on wage setting ensure that outside offers do not matter for the joint continuation value.

This cannot be done with risk averse workers and income taxes. The wage is not a one-for-one transfer between workers and firms. Therefore, we cannot just use the surplus function, but have to solve for the value functions and wages simultaneously.

Distributions. Besides the value functions, the other key objects to be solved for are four distributions. These are the distribution of unemployed workers $\mu_x(x)$, the distribution of vacant jobs $\mu_y(y)$, the distribution of matches at the production stage $\psi(x,y)$, and the distribution of matches at the search stage $\psi^S(x,y)$. Note that in principle the distributions of matches could have an additional dimension in form of the wage. However, in this setup, the only thing that matters for determining transitions and for which wages have to be paid in case of job-to-job transitions and outside offers is how much employers can potentially pay, which is pinned down by the productivities. It does not matter what the current wage is. Therefore, we can omit wages as an argument from these distributions.

Consider first the distribution of matches at the production stage. This is updated based on inflows from unemployment, inflows from employment, and retained matches.

$$\begin{aligned}\psi(x,y) = {} & \lambda_u A_U(x,y)\frac{\mu_y(y)}{v}\mu_x(x) \\ & + \lambda_e \int_{\tilde{y}} A_E(x,\tilde{y},y)\,\psi^S(x,\tilde{y})\,d\tilde{y}\frac{\mu_y(y)}{v} \\ & + \left(1 - \lambda_e \int_{\tilde{y}} A_E(x,y,\tilde{y})\frac{\mu_y(\tilde{y})}{v}d\tilde{y}\right)\psi^S(x,y)\end{aligned} \tag{1.12}$$

An unemployed worker of productivity x from the distribution $\mu_x(x)$ meets a vacant job with probability λ_u. This vacant job is of type y with probability $\mu_y(y)/v$. A match between these productivity types is created with probability $A_U(x,y)$. Note that productivity shocks do not have to be taken into account here. Timing is such that productivity shocks happen between the production and the search stage. Therefore, the matches created at the search stage between unemployed workers and vacant jobs enter the distribution of matches at the production stage with their respective productivities. This case is taken care of in line 1 of Equation (1.12).

Line 2 deals with inflows from employment. A worker of type x can be employed at any firm productivity type $\tilde{y}$ according to the distribution of matches at the search stage ψ^S. This worker will become part of $\psi(x,y)$ because of a job-to-job transition if there is a meeting (probability λ_e), the meeting is with a vacancy of type y (probability $\mu_y(y)/v$), and the match is created with this new firm, which happens with probability $A_E(x,\tilde{y},y)$. Again, no productivity shocks happen between the creation of these matches and the production stage.

Line 3 accounts for retained matches. A match at the search stage stays in place for the next production stage if the worker does not meet a vacant job leading to a job-to-job transition. Also here, it is not necessary to account for productivity shocks.

The second important distribution is the distribution of matches at the search stage. This differs from the distribution of matches at the production stage because of exogenous separations, productivity shocks, and endogenous separations following productivity shocks:

$$\psi^S\left(x',y'\right) = (1-\delta)\int_x\int_y A_S\left(x',y'\right)\psi(x,y)\,p\left(y' \mid y\right)p\left(x' \mid x\right)dydx. \tag{1.13}$$

The third endogenous distribution is the distribution of vacant jobs. It is computed from newly created jobs, unfilled matches, and filled matches that become vacant because of either endogenous separations or job-to-job transitions.

$$\begin{aligned}
\mu_y\left(y'\right) = &\int_y (1-\delta)\,v^N(y)\,\phi_y(y)\,Mp\left(y' \mid y\right)dy \\
&+(1-\delta)\int_y\Bigg\{(1-\lambda_f)+\lambda_f\Big\{p^u\int_x\left(1-A_U(x,y)\right)\frac{\mu_x(x)}{u}dx \\
&+(1-p^u)\int_{\tilde{y}}\int_x\left(1-A_E(x,\tilde{y},y)\right)\frac{\psi^S(x,\tilde{y})}{e^S}dxd\tilde{y}\Big\}\Bigg\}p\left(y' \mid y\right)\mu_y(y)\,dy \\
&+\lambda_e(1-\delta)\int_y\int_x\int_{\tilde{y}}A_E(x,y,\tilde{y})\frac{\mu_y(\tilde{y})}{v}\psi^S(x,y)\,p\left(y' \mid y\right)d\tilde{y}dxdy \\
&+\int_{y^-}\int_{x^-}(1-\delta)^2\left(1-A_S(x,y)\right)\psi\left(x^-,y^-\right)p\left(y' \mid y\right)p\left(x \mid x^-\right)p\left(y \mid y^-\right)dx^-dy^-.
\end{aligned} \tag{1.14}$$

Line 1 of Equation (1.14) accounts for newly created jobs. Newly created jobs are denoted with $v^N(y)$. A firm with productivity y creates new vacant jobs until the marginal cost of doing so equals the value of a vacant job:

$$
\begin{aligned}
&c'\left(v^N(y)\right) = V(y), \\
&\Rightarrow v^N(y) = c'^{-1}(V(y)).
\end{aligned}
\tag{1.15}
$$

New jobs are created at the beginning of the period, so they can be exogenously destroyed. New job creation is weighted with the total mass of firms M and the exogenous distribution of firms across productivities $\phi_y(y)$. Also, between the production stage and the search and matching stage productivity shocks take place.

The second and third lines of Equation (1.14) deal with unfilled vacant jobs surviving from the period before. To still be part of the distribution of vacant jobs, a job may not be exogenously destroyed. Given this is the case, there are three scenarios in which a job remains vacant. First, it may not meet any worker. Second, it may meet an unemployed worker, but productivities are such that no match is created. Third, it may meet an employed worker, but productivities are such that there is no job-to-job transition.

Lines 4 and 5 take care of separations that cause a previously filled job to become vacant. This can happen either because of a job-to-job transition or an endogenous separation. Consider first job-to-job transitions (line 4). Starting from the distribution of matches at the search stage, a worker may meet a vacant job and do a job-to-job transition. Then, the firm will have a vacant job. Until the search stage of the next period, however, this job can be exogenously destroyed and is subject to the firm productivity shock.

Consider next endogenous separations (line 5). Starting from the distribution of matches at the production stage, jobs can be exogenously destroyed. In that case they do not enter the distribution of vacant jobs. Those matches that survive exogenous destruction shocks are subject to productivity shocks. After these realize, there may be endogenous separations. However, the vacant jobs can only be refilled at the search stage of the next period, so that they are subject to exogenous destruction and productivity shocks again before entering the distribution of vacant jobs.

Finally, the distribution of unemployed follows as the difference between the exogenous distribution of worker types and the workers in employment:

$$
\mu_x(x') = \int_x \left[\phi_x(x) - \int_y \psi(x,y)\,dy\right] p\left(x' \mid x\right) dx. \tag{1.16}
$$

This is due to the assumption that those who are separated cannot search within the same period. Therefore, they will show up in the distribution of unemployed only in the next period. However, productivity shocks apply between the production stage and the search stage.

1.3.2 Calibration

We calibrate the model to the Danish labor market. A model period is a month.

Functional Forms. For solving the model, we have to specify a number of functional forms. We start with the utility function. We assume that workers have a standard constant relative risk aversion utility function

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma} \tag{1.17}$$

with coefficient of relative risk aversion γ.

Match output has the following functional form:

$$f(x,y) = \nu \left(x^{\frac{1}{\rho}} + y^{\frac{1}{\rho}} \right)^{\rho}, \tag{1.18}$$

where ν governs the level of production and ρ the complementary between worker and firm productivity in production, which is important for sorting.

The matching function takes the standard Cobb-Douglas functional form:

$$M(L,v) = \xi L^{\omega} v^{1-\omega}. \tag{1.19}$$

This functional form is widely used in the labor literature and is consistent with the data (Petrongolo and Pissarides, 2001).

The job creation cost function is given by

$$c(v) = c_0 \left(\frac{v}{c_1} \right)^{c_1}, \tag{1.20}$$

as in Merz and Yashiv (2007), Bagger and Lentz (2019), and Gulyas (2020), among many others.

To model the progressive income tax scheme, we rely on the widely used loglinear income tax function:

$$T(w) = w - \lambda w^{1-\tau}, \tag{1.21}$$

which has been popularized by Feldstein (1969), Benabou (2002), and Heathcote, Storesletten, and Violante (2017). With this tax function, λ governs the level of tax rates, whereas τ determines the progressivity. With positive τ, average and marginal tax rates are increasing in income. $\tau = 0$ implies a flat tax rate, whereas a negative τ means that the tax system is regressive.

Finally, we have to specify the stochastic processes for worker and firm productivity. We assume that worker productivity follows an AR(1) in logs:

$$\log x_t = \rho_x \log x_{t-1} + \eta_x, \quad \eta_x \sim \mathcal{N}\left(0, \sigma_x^2\right). \tag{1.22}$$

Firm productivity remains the same with probability $1 - \pi_y$. With probability π_y, a firm draws a new productivity. This new productivity level can be drawn from a range around the old productivity level. The size of this range is governed by the parameter $\bar{y}$. This is a common way of modeling productivity shocks in the firm dynamics literature (Kaas and Kircher, 2015).

Targeted Moments and Parameters. We parameterize the model in two steps. First, we set a number of parameters externally. Second, we choose the remaining parameters to match key properties of the Danish labor market.

In the first step, we exogenously fix preference parameters. We set the discount factor β to 0.995, corresponding to an annualized discount factor of 0.94. We set the coefficient of relative risk aversion γ to two.

In the Danish registry data we observe taxes paid by an individual, so we can compute pre- and after-tax income. We use this to estimate the parameters of the loglinear income tax function. To do so, we regress after-tax income on pre-tax income as in Heathcote, Storesletten, and Violante (2017). We estimate the coefficients by year, as taxes are based on annual income. The coefficients are fairly stable over time. Our estimate for λ is 0.92 and the estimated progressivity τ is 0.22. The estimated τ is much higher than in many other countries, reflecting the high progressivity of the Danish tax system. This is in line with previous estimates such as Holter, Krueger, and Stepanchuk (2019), who estimate a progressivity parameter of 0.258 for Denmark using OECD data. For simplicity we apply the tax function to monthly income. Applying it to annual income would require introducing another state variable keeping track of previous income during the year. We fix the unemployment benefit at a constant level. It does not expire, which is a reasonable approximation since the Danish unemployment insurance system is very generous and benefits are available for up to three years. There are no layoff taxes or vacancy subsidies, so we set them to zero in the calibration.

For the labor market parameters, we exogenously fix the worker bargaining power α when being unemployed to 0.2. We set the curvature of the matching function ω to 0.5, as in Petrongolo and Pissarides (2001). The remaining parameters are calibrated internally. The matching function level

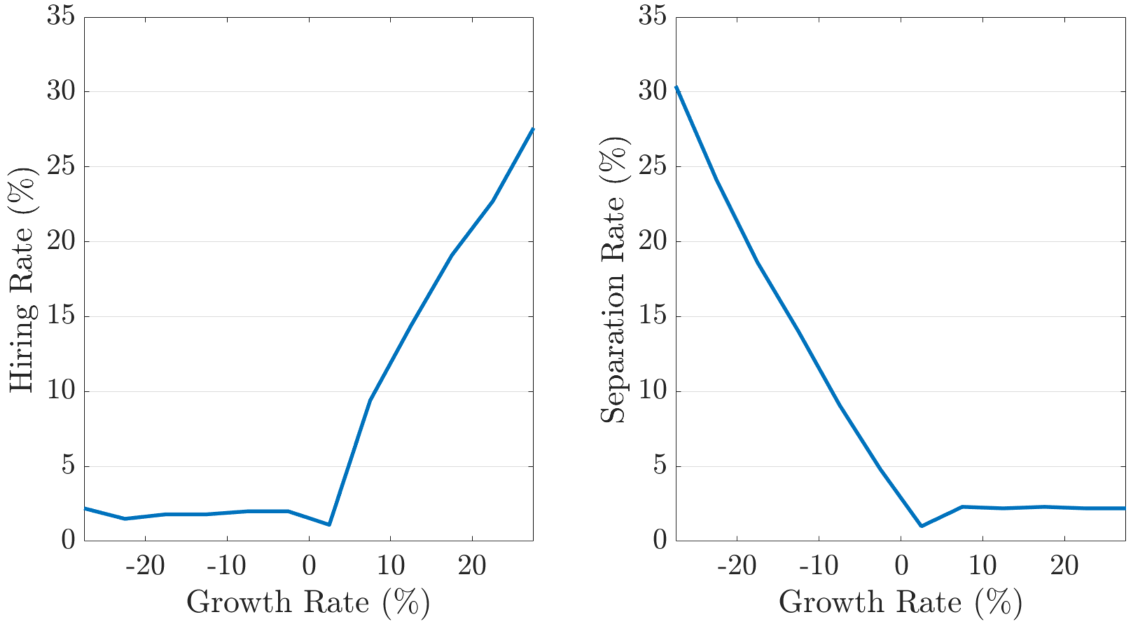


Figure 1.10 Hiring and Separation Rates

Notes: This figure shows firm hiring and separation rates by firm growth rate.

parameter ξ is closely associated with the transition rate from unemployment to employment and thereby the unemployment rate, which averaged around 6% in the time period we are interested in. Search intensity on the job s is set to match a monthly job-to-job transition rate of 1.7%. To calibrate the exogenous job destruction rate we compute hiring and separation rates as a function of firm employment growth rates. These are shown in Figure 1.10. Hiring and separation rates follow the hockey stick shape that is also present in U.S. data (Davis, Faberman, and Haltiwanger, 2013; Mongey and Violante, 2019). For negative growth rates, the hiring rate is flat at around 1.5-2%. For positive growth rates, the hiring rate is monotonically increasing. The separation rate is decreasing for negative firm growth rates and flat at around 2% for positive growth rates. The stable positive separation rate for growing firms in particular is informative for the exogenous job destruction parameter δ. On the worker side, the monthly employment to unemployment transition rate is 2.6%.

The model does a very good job at matching the unemployment rate and the average job-to-job transition rate. The unemployment rate by worker productivity level is shown in Figure 1.11. The unemployment rate is constant for high productivity levels. This is the case because the model is a random search model, so that meeting probabilities do not differ by productivity. Matches with high productivity workers will always be created and never be dissolved voluntarily, so that the unemployment rate for these groups is the same. At low productivity levels, however, the unemployment rate is higher.

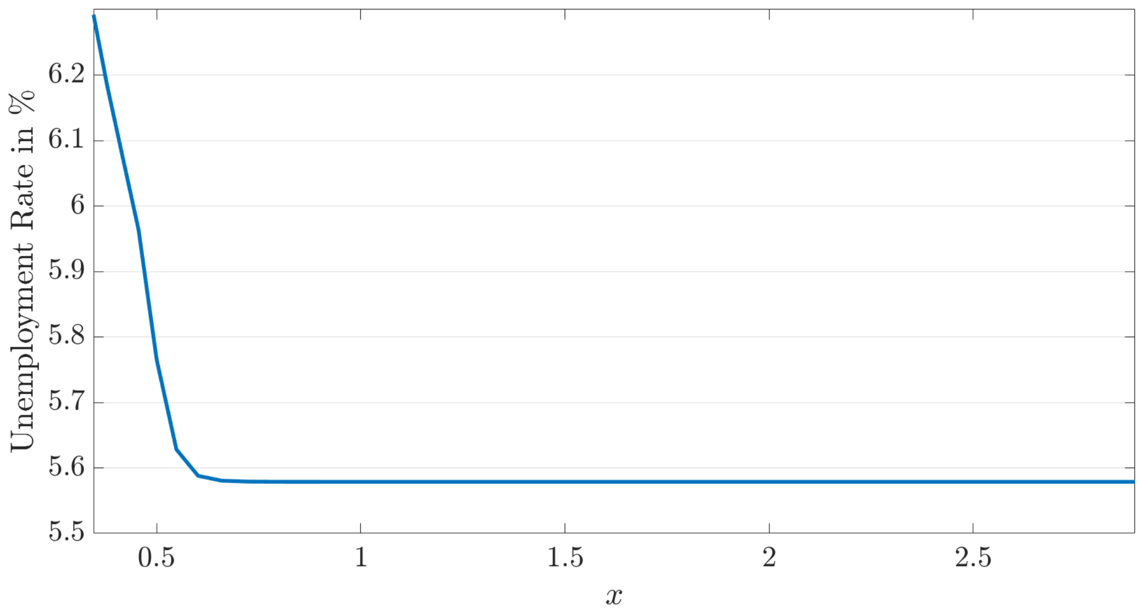


Figure 1.11 Unemployment Rate

Notes: This figure shows the unemployment rate by worker productivity level.

While these workers when searching as unemployed have as many meetings as more productive workers because of random search, not all these meetings will lead to matches. Also, when worker productivity falls to these low levels, depending on firm productivity, an existing match could be split up. The model does, however, understate in the current calibration the average EU rate. Even after negative productivity shocks it is too often feasible to renegotiate and keep matches alive rather than endogenously separating, which occurs only relatively rarely.

The worker productivity parameters are set to match the wage distribution. For that purpose we compute moments of the monthly earnings distribution from the data. The model does fairly well in capturing income inequality for most parts of the earnings distribution. For example, mean earnings of the 90th to the 95th percentile of the earnings distribution relative to mean earnings of individuals between the 5th and the 10th percentile of the earnings distribution is 3.39 in the data and 3.49 in the model. However, while this value is even slightly too high, the model fails to match the levels of income at the very top: The model does not have a mechanism to generate the very large incomes of the very top earners.

Finally, the parameters for the firm productivity process and the job creation cost function are closely related to the firm size distribution. Mean firm size in the data is 19. The firm size distribution is extremely skewed with the 75th percentile being 8, the 90th percentile being 20, and the 95th

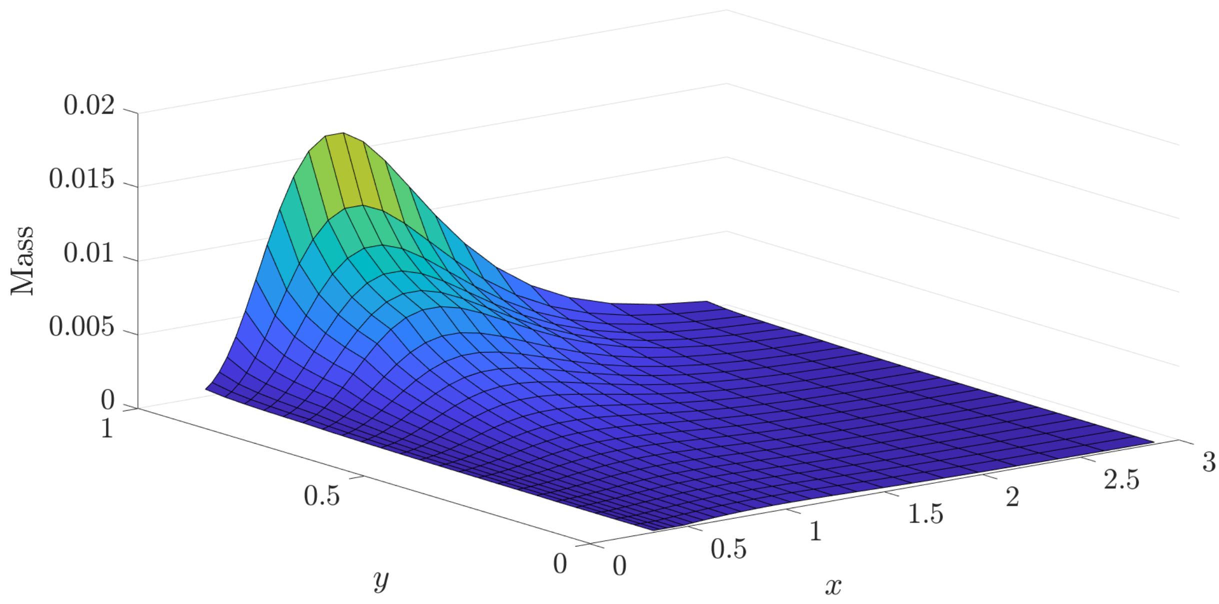


Figure 1.12 Distribution of Matches

Notes: This figure shows the distribution of workers across firm productivity levels.

percentile being 167. The model matches average firm size almost perfectly, but does not yet produce the extreme concentration of the workforce in the largest firms. This is a common problem in firm dynamics models, which could be fixed for example with firms being heterogeneous in two dimensions, one permanent productivity type and an additional stochastic productivity component. Permanent productivity differences are very powerful in generating skewed firm size distributions; see for example the model of Kaas and Kircher (2015), which features a permanent and a transitory firm productivity component. To economize on state variables, we only incorporate a transitory component and therefore miss to some extent the extreme concentration of the workforce in the very largest firms.

An important parameter that we do not take a stand on so far is the complementarity parameter ρ, which we set to one. This is to be disciplined carefully in the next version. All model parameters are summarized in Table 1.1.

Figure 1.12 shows the model implied distribution of matches at the production stage. It is apparent from the figure that a large share of matches is concentrated in the high productivity firms. Matches are more valuable for more productive firms, who will therefore post a larger number of vacant jobs. They also fill these jobs more quickly because very productive firms form matches with all worker types and because they can poach employed searchers from less productive firms. There is no mass at the combinations of low worker and low firm productivity as these matches are not feasible.

Table 1.1 Parameter Values

Parameter	**Interpretation**	**Value**
Preferences		
β	Discount factor	0.9950
γ	Risk aversion	2.0000
Labor market		
α	Bargaining power	0.2000
ξ	Matching function level	0.2000
ω	Matching function curvature	0.5000
s	Search intensity on-the-job	0.1200
δ	Job destruction probability	0.0200
Workers		
ρ_x	Worker productivity persistence	0.9900
σ_x	Worker productivity std. dev.	0.0500
Firms		
π_y	Prob. of new firm prod. draw	0.1000
$\bar{y}$	Firm prod. adjustment range	5.0000
M	Mass of firms	0.0420
Production		
ν	Production function scale	1.2000
ρ	Production complementarity	1.0000
Job creation		
c_0	Job creation cost level	22.0000
c_1	Job creation cost curvature	1.0500
Government		
b	Unemployment benefit	0.3000
λ	Income tax level	0.9200
τ	Income tax progressivity	0.2200
τ_L	Layoff tax	0.0000
τ_V	Vacancy subsidy	0.0000

Notes: Table 1.1 summarizes the parameter values.

1.3.3 Model Results

We now turn to the model implications for the untargeted distributions of worker earnings growth, firm revenue growth, and worker earnings growth conditional on firm revenue growth. As in the data, we start with the distribution of worker earnings growth.

The model captures very well the key feature of the earnings growth distribution, which is excess kurtosis, as shown in Figure 1.13. There is significant mass around zero earnings: The model produces this because wages are only renegotiated or even matches dissolved if very bad productivity shocks occur. For most small shocks noone has a credible threat to leave the match, so earnings are unchanged. The model also captures through this mechanism that the earnings growth distribution has fat tails. If productivity shocks are extreme, this will trigger a renegotiation potentially leading to very large earnings changes. Large earnings drops are also generated through exogenous separations, while large earnings gains are also generated through renegotiations after outside offers and job-to-job transitions.

Figure 1.14 shows the annual firm revenue growth distribution in the model. Also for this distribution the model captures the excess kurtosis very well. It also captures that while there is a lot of mass around zero revenue changes the distribution is less concentrated around zero than for worker earnings. The reason why there is a lot of mass around zero is that firms are only likely to voluntarily separate from workers if productivity falls very low. Also, the posting of new vacancies is very skewed towards firms with very good productivity realizations.

Finally, in Figure 1.15 we show the earnings growth distribution conditional on firm revenue growth being between -0.30 and -0.25 or between 0.25 and 0.30, which corresponds to the groups we already looked at in the empirical part. This is constructed, as in the data, conditioning on a worker being employed for the entire twelve months in the base year. The model does capture that the worker earnings growth distribution has more mass in the left tail for workers employed at shrinking firms. This is driven by endogenous separations in the firms experiencing shocks to the very worst productivity levels and renegotiations of wages also in firms where productivities drop at higher levels. The model also reproduces that there is a lot of mass at zero for workers in shrinking and in growing firms. Finally, the model reproduces a longer right tail of the earnings growth distribution for workers in growing firms. Firms that grow tend to have positive productivity shocks and therefore also higher levels of productivity. In such firms, outside offers are more valuable because the firm can afford to pay for larger earnings gains.

In Figure 1.16 we additionally condition on the worker being employed for twelve months in the second year at the same firm. As in the data, the earnings growth distribution of this subsample has

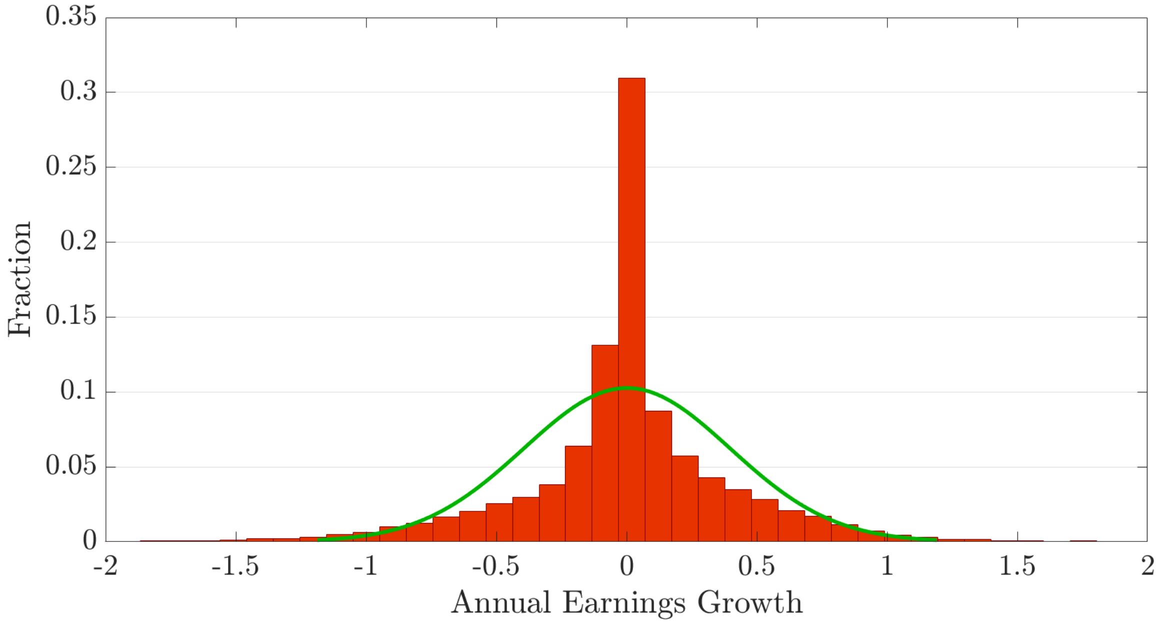


Figure 1.13 Model: Annual Earnings Growth Distribution

Notes: This figure shows the annual worker earnings growth distribution in the model.

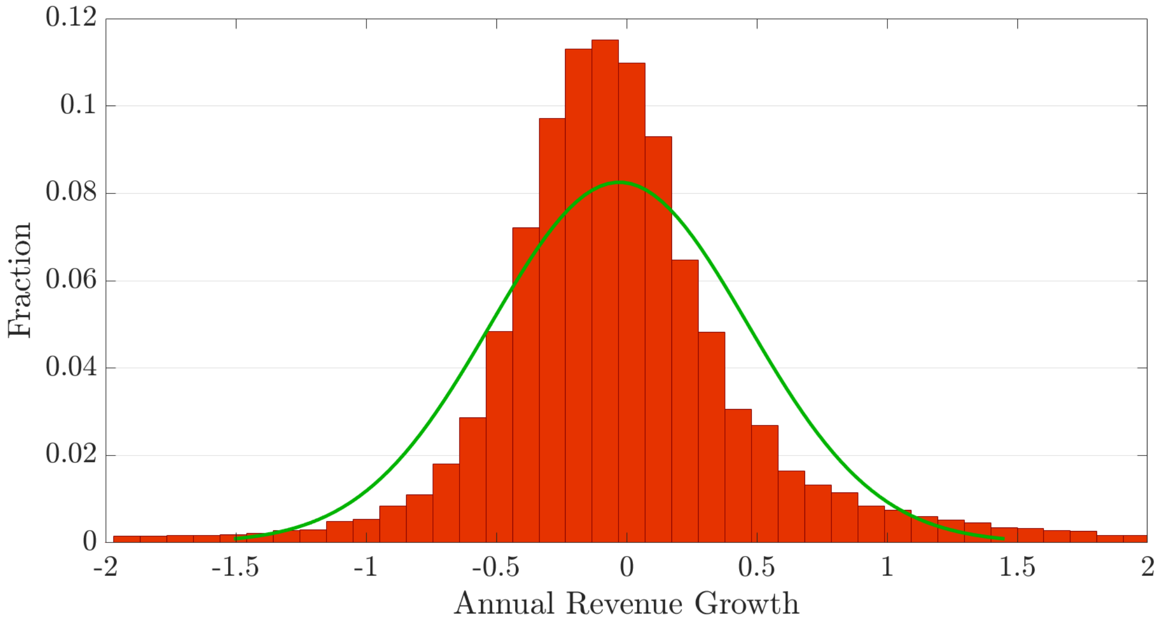


Figure 1.14 Model: Annual Firm Revenue Growth Distribution

Notes: This figure shows the annual firm revenue growth distribution in the model.

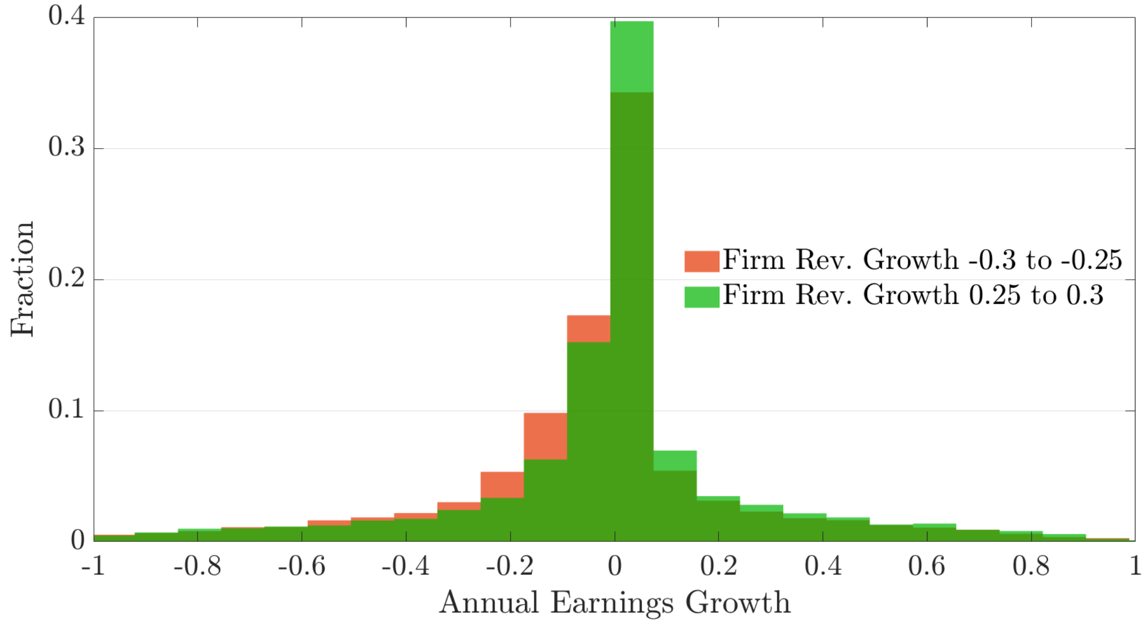


Figure 1.15 Model: Worker Earnings Growth by Firm Revenue Growth

Notes: This figure shows the worker earnings growth distribution conditional on firm revenue growth.

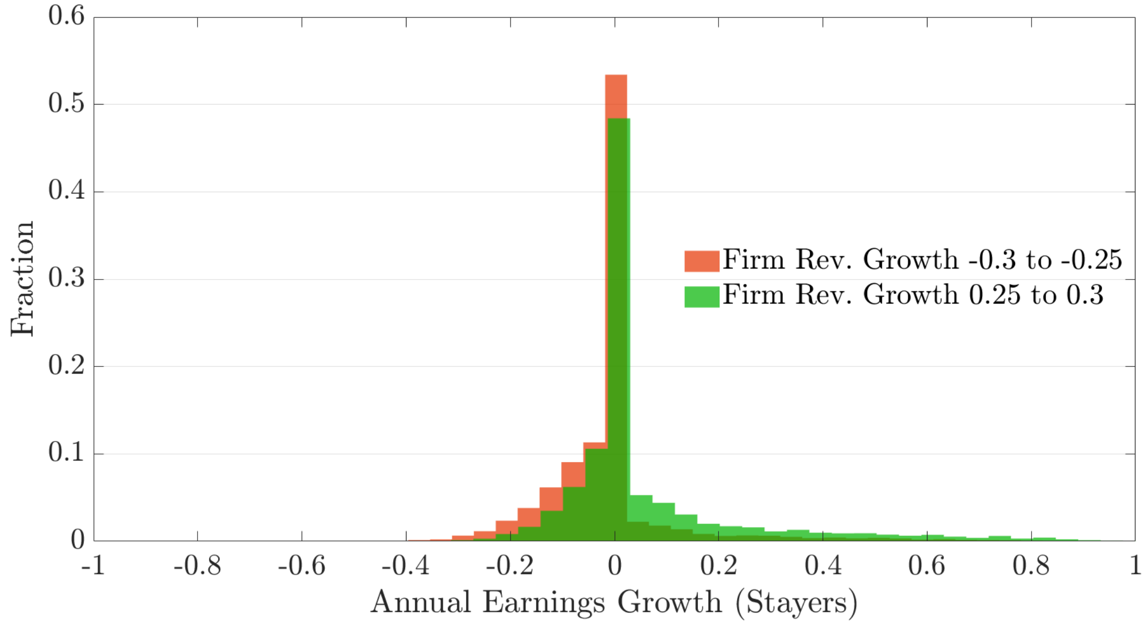


Figure 1.16 Model: Worker Earnings Growth by Firm Revenue Growth (Stayers)

Notes: This figure shows the worker earnings growth distribution of job stayers conditional on firm revenue growth.

much lower variance compared to the previous case because there is noone included whose earnings vary strongly because of unemployment spells. Still, higher firm growth is associated with a shorter left and a longer right tail of the worker earnings growth distribution because of renegotiations after productivity shocks and outside offers.

Preliminary Policy Experiments. We use the model to conduct a couple of preliminary exploratory policy experiments. In principle, the model naturally lends itself to a comparison of policies stabilizing worker earnings directly at the worker side (progressive taxes, unemployment benefits) or targeting firms (layoff taxes, vacancy subsidies). Here, we only discuss the trade-offs associated with two of these policies, which are progressive taxes and vacancy subsidies.

Progressive taxes have a very immediate strong effect on the dispersion in after-tax earnings and earnings changes. With more progressive taxes, after-tax inequality is significantly reduced and large pre-tax earnings changes translate into smaller after-tax earnings changes. However, a standard efficiency-redistribution trade-off emerges. Very progressive taxes lower the surplus especially for high productivity matches. This discourages vacancy posting by the most productive firms, which leads to more unemployment in equilibrium and a lower share of workers employed at the highest productivity firms.

Vacancy subsidies, by contrast, make the creation of vacant jobs cheaper, which stimulates vacancy posting. In equilibrium, there is a lower unemployment rate and a higher share of employed workers working at the most productive firms. However, this is a costly policy which needs to be financed by the government. In future versions, we will compare these and other policies more rigorously.

1.4 Conclusion

In this paper we provide evidence from Denmark on worker earnings risk, firm dynamics, and their relationship. Both the worker earnings growth distribution and the firm revenue distribution are characterized by excess kurtosis: Most workers/firms experience very small changes to their earnings/revenues, but a significant share of workers/firms sees their earnings/revenues change drastically. The majority of workers in shrinking and growing firms alike experiences small earnings changes, but the likelihood of very large earnings drops is significantly higher in shrinking firms. This is driven by separations to unemployment and wage drops for continuously employed workers.

We then develop a model of multi-worker firms operating in frictional labor markets that features risk averse workers searching on- and off-the-job and contracting with two-sided limited commitment. The model, when calibrated to the Danish economy, is consistent with the key patterns found in the data. Preliminary policy experiments suggest that policies targeting workers or firms can be used to stabilize earnings, which is valued by risk averse workers, while creating different efficiency costs.

In the next steps, we are going to estimate the model on Danish data and perform a more rigorous policy analysis. Also, the worker earnings growth distribution, firm revenue growth distribution, and the joint distribution exhibit important cyclical patterns. We plan to add aggregate risk in the form of an MIT shock to the model in order to investigate whether the model is also consistent with the evidence over the business cycle.

Chapter 2

Joint Search over the Life Cycle

Abstract This paper studies how the added worker effect - intra-household insurance through increased spousal labor market participation - varies over the life cycle. We show in U.S. data that the added worker effect is much stronger for young than for old households. A stochastic life cycle model of two-member households with job search in a frictional labor market is capable of replicating this finding. The model suggests that a lower added worker effect for the old is driven primarily by better insurance through asset holdings. Human capital differences between employed young and old contribute to the difference but are quantitatively less important, while differences in job arrival rates play a limited role.

2.1 Introduction

Household earnings dynamics vary strongly over the life cycle. Recent literature documents that key moments of the earnings growth distribution exhibit significant age-dependency (De Nardi, Fella, and Paz-Pardo, 2019; Guvenen, Karahan, Ozkan, and Song, 2021). Earnings variability is highest for young individuals as they change jobs frequently before settling into a stable job. However, the earnings growth distribution is more left-skewed for older individuals: Most of the time older individuals are employed in stable employment relationships at relatively high wages. If they lose this job, however, this fall off the job ladder implies very large earnings losses. In this paper we take a complementary perspective: Instead of investigating how risks change over the life cycle, we study how insurance against individual earnings risk varies over the life cycle. Specifically, we focus on an insurance margin against individual earnings and unemployment risk available to couples, the added worker effect (AWE), where a previously non-participating spouse enters the labor force upon job loss of the primary earner to stabilize joint earnings.

While the added worker effect has in general been widely documented, our focus on how it varies over the life cycle is novel to the literature.[1] Age differentials in the AWE are important for a variety of reasons: Observed heterogeneity along this margin improves our understanding of how well households at different ages are insured against income losses. Therefore, disparities in the availability of this self-insurance margin can alter the optimal provision of public insurance over the life cycle. Moreover, in light of demographic change any difference in the labor market behavior of old versus young households can change aggregate labor market dynamics.

We begin by providing empirical evidence on the added worker effect over the life cycle: Using data for the United States from the Current Population Survey (CPS), we show that the likelihood of a non participating spouse entering the labor force increases significantly when the primary earner loses her job compared to when she remains employed. We find, however, a strong age-dependency in this effect. In particular, the added worker effect is largest for young households and continuously declines over the life cycle. For the age group just before retirement, the added worker effect is almost non-existent. For young households, job loss of the primary earner is associated with a significant increase in the likelihood of an out of the labor force spouse entering the labor force both directly to employment and to unemployment. This finding is robust across education levels, the presence of children in the household, different reasons for being out of the labor force, different reasons for an employment to unemployment transition of the primary earner, and looking at only one cohort.

Still, there remain several candidate explanations for the observed change in the AWE over the life cycle. It might be that older households have accumulated sufficient asset holdings that allow them to smooth consumption during a potentially temporary job loss of the primary earner. In this case, older households do *not need* the added worker effect as an (additional) insurance margin. An out of the labor force spouse could in principle join the labor force, find employment, and stabilize joint earnings, but chooses not to do it. Alternatively, it could be that older spouses have been out of the labor force for a long time such that their labor market qualifications have become less valuable than those of younger individuals. In this case, spousal labor supply is *unavailable* as an insurance margin if the spouse can provide little marketable skills. In order to distinguish between the *need for* and the *availability of* the spousal insurance margin, we build a quantitative model of joint labor supply over the life cycle in a frictional labor market.

In the model, a household consists of two members, each of whom can be either employed, unemployed (and actively searching for a job), or out of the labor force. The labor market is frictional, an individual can only take up employment if she has a job offer. While both out of the labor force and

[1] See the related literature below for a detailed discussion.

unemployed individuals can receive job offers, unemployed members increase the chance of finding a job through costly search. Employed individuals face the risk of (exogenous) separation and wage changes due to match quality shocks. Human capital is accumulated while employed but depreciates during non-employment and the couple can jointly save in a risk-free bond. Job arrival rates are endogenous and determined by the solution to a vacancy posting problem of single-worker firms.

These model ingredients allow us to differentiate between the potential different explanations for the age dependency in the added worker effect. Household savings are a key alternative insurance mechanism against individual unemployment risk. With a realistic life cycle savings profile the model can speak to whether differences in asset holdings between young and old are sufficient to explain the difference in the observed AWE. On the other hand, human capital accumulation and endogenous arrival rates allow for the possibility that older households might have fewer opportunities to provide insurance against individual risk, as human capital depreciates over long spells out of the labor force. Furthermore, firms might be less willing to hire older individuals as there is only little time remaining to recover hiring costs. These two model elements capture that older individuals might have fewer opportunities to provide insurance against a spouse's job loss.

We calibrate the model to match key features of the U.S. labor market and of inequality over the life cycle. For the labor market, we focus on matching average transition rates across labor market states as well as the joint distribution of couples across labor market states. For inequality, we match life cycle income profiles and asset holdings over the life cycle. Without targeting them, the model reproduces reasonably well life cycle profiles of labor market transitions as well as very closely the age-dependency in the added worker effect. The model captures very well that the effect is largest for the young and smallest for the age group just before retirement.

With the calibrated model at hand, we perform counterfactuals to evaluate which mechanisms are important in explaining the age-dependency in the added worker effect. Our results suggest a significant influence of larger asset holdings of older households, which can serve as a cushion against temporary job loss. Higher human capital levels of old employed spouses relative to their younger counterparts – accumulated during a longer working life – make spousal labor supply less valuable as an insurance margin but are quantitatively less important. Differences in job arrival rates for young and old out of the labor force spouses play a limited role, as they turn out to be relatively low for both age groups.

In future work, we will evaluate the consequences of these mechanisms for the provision of optimal life cycle unemployment insurance. For such an analysis it is key to match the risk exposure of households over their life cycle as well as the private insurance mechanisms, which could be crowded out through public transfer payments. As our model covers a wide range of insurance mechanisms available to households at different stages of their life cycle, the framework naturally lends itself to this question. Michelacci and Ruffo (2015) study optimal life cycle unemployment insurance using a single earner life cycle search model.[2] They argue that unemployment insurance should be more generous for the young than for the old, as the insurance value is very high for individuals with little assets and the moral hazard problem is limited, as young individuals need to accumulate labor market experience. Studying this question in a search model of couples is relevant because unemployment insurance could crowd out the added worker effect, which is an important insurance margin for the young.

Related Literature. The added worker effect is widely studied in the empirical literature, going back to the seminal contribution of Lundberg (1985). The early literature following this paper does not find much evidence supporting the presence of the added worker effect in the data (Maloney, 1987, 1991). More recent literature, however, documents a positive added worker effect as a relevant insurance mechanism against the primary earner's job loss (Bredtmann, Otten, and Rulff, 2018; Guner, Kulikova, and Valladares-Esteban, 2020; Halla, Schmieder, and Weber, 2020; Stephens, 2002), using data for a wide variety of countries. Mankart and Oikonomou (2016b) and Mankart, Oikonomou, and Pascucci (2021) show that the added worker effect has become more important in the U.S. over the last decades. The literature also argues that the size of the added worker effect crucially depends on the institutional environment and the state of the business cycle. For example, Cullen and Gruber (2000) show that generous unemployment insurance crowds out a spousal labor supply response. Expanding upon previous work, we argue that there is a sizeable age-dependency in the added worker effect.

While the added worker effect has been studied extensively in the empirical literature, the vast majority of the large macro-labor literature focuses on the job search problem of a single earner household. Guler, Guvenen, and Violante (2012) is among the first papers to study the joint search problem of a couple by extending the classic single-agent search problems of McCall (1970), Mortensen (1970), and Burdett (1978). A number of recent papers introduces asset accumulation into the joint search framework, expanding on the single agent search problem with asset accumulation as in Lentz (2009), Krusell, Mukoyama, and Şahin (2010), and Krusell, Mukoyama, Rogerson, and Şahin (2017). The focus of these papers is mostly on business cycle dynamics. Mankart and Oikonomou (2016a) build a search model with two member households to explain the cyclical properties of employment and labor force participation. Wang (2019) builds a model showing that joint household search is crucial for accounting for the countercyclicality of womens' unemployment rate. Ellieroth (2019) argues that there

[2] Optimal age-dependent policies are also commonly studied in public finance. See for example Erosa and Gervais (2002), Weinzierl (2011), and Heathcote, Storesletten, and Violante (2020a).

is precautionary labor supply by spouses whose partners face an increased job loss risk in recessions. Garcia-Perez and Rendon (2020) focus on the role of household wealth for the added worker effect. Birinci (2019), Choi and Valladares-Esteban (2020), and Fernández-Blanco (2020) investigate the implications of joint search for optimal unemployment insurance. Bardóczy (2020) focuses on the role of spousal labor supply as an automatic stabilizer for aggregate consumption. Relative to these papers, we focus on the life cycle dimension of the joint search problem to analyze whether the age-dependency in the added worker effect is explained by changing opportunities or changing insurance margins.

Life cycle search problems have been studied in the literature, but mostly in single earner frameworks. Chéron, Hairault, and Langot (2011, 2013) extend the random search framework of Mortensen and Pissarides (1994) to a life cycle setting. Menzio, Telyukova, and Visschers (2016) build a directed search life cycle model in the tradition of Moen (1997) and Menzio and Shi (2011). Griffy (2021) extends their model by incorporating risk averse workers and borrowing constraints. More closely related to our paper, Haan and Prowse (2017) propose a structural life cycle model of labor supply, consumption, and savings of married couples. They focus on the optimal mix of unemployment insurance and social assistance but do not discuss any age-dependency in the added worker effect. Finally, the current paper is related to a number of studies analyzing life cycle labor supply decisions of couples in incomplete market frameworks (Blundell, Pistaferri, and Saporta-Eksten, 2016; Ortigueira and Siassi, 2013; Wu and Krueger, 2021).

Roadmap. The paper proceeds as follows. Section 2.2 contains the empirical evidence. In Section 2.3 we introduce the model setup. Section 2.4 contains the calibration and section 2.5 the results. Section 2.6 concludes.

2.2 Evidence

We begin by providing evidence on the added worker effect from U.S. micro data. The following section first explains the data and the sample selection criteria. In a next step, we provide empirical evidence of the AWE in our sample and show that its magnitude is decreasing in age.

2.2.1 The Sample

To compute joint labor market transitions, we work with data from the Current Population Survey (CPS). The CPS is a monthly rotating panel which is representative for the U.S. population. Households enter the survey for four consecutive months, drop out for eight months, and are re-interviewed for another four months. In our setting, the unit of observation is a couple. Our final sample spans from 1994 until 2020 (pre-Covid) and is restricted to couples who are both between 25 and 65 years old. We mainly focus on couples with one spouse working and the other spouse out of the labor force. We include both legally married as well as cohabiting couples, irrespectively of their sex. In contrast, we drop couples who report that one spouse lives permanently outside of the household or is institutionalized. Moreover, we only keep couples for whom we observe the labor market status of both spouses in every month that they are interviewed. Throughout the analysis, we weigh each observation by the provided survey weights.

2.2.2 Uncovering the AWE from Joint Labor Market Transitions

We follow Guner, Kulikova, and Valladares-Esteban (2020) in our method to calculate the added worker effect from the data. First, we classify all individuals either as *employed* (E), *unemployed* (U) or *non-participating* (N) as outlined in the CPS. Hence, there exist nine possible combinations of labor market states for each couple. A common issue when considering multiple non-employment states is misclassification between unemployment and non-participation, resulting in implausibly high transition rates across these two. We therefore adjust labor market flows as in Elsby, Hobijn, and Şahin (2015) and re-classify individuals who report to be unemployed (non-participating) in one month but to be out of the labor force (unemployed) in both the following and in the previous month as non-participating (unemployed).

In a next step, we pool all observations and construct a 3×3 matrix of joint labor market transition probabilities, conditional on the couple having one member previously employed and one out of the labor force. Table 2.1 and Table 2.3 display our main results. In each table, the columns refer to the monthly labor market transition of the household's primary earner, that is either employment-to-employment (EE), employment-to-unemployment (EU), or employment-to-non-participating (EN). In contrast, each row indicates the probability of the spousal labor market transition, conditional on the respective transition of the primary earner. Given that for this exercise we only include couples with one member employed and the other one non-participating, spouses can either transition from non-participating to employment (NE), from non-participating to unemployment (NU) or remain out of the labor force (NN). We define the added worker effect as the change in the conditional probability of the spouse transitioning from non-participating to employment (NE) or from non-participating to unemployment (NU) if the primary earner becomes unemployed (EU) in contrast to when the primary earner remains employed (EE). Referring to Table 2.1, we compute the added worker effect as the difference between the second and first column, adding up the first and the second row.

Table 2.1 Joint Labor Market Transitions (Full Sample)

	Primary earner transition EE	EU	EN
Cond. prob. of spousal NE transition	6.03%	8.01%	16.79%
Cond. prob. of spousal NU transition	1.63%	5.55%	1.33%
Cond. prob. of spousal NN transition	92.34%	86.44%	81.88%

Notes: This table shows the probability of a spousal transition from out of the labor force conditional on primary earner transitions for the entire population.

Overall Effect

Table 2.1 shows the overall strength of the added worker effect in our sample. The likelihood that a spouse enters the labor force increases by 5.9 percentage points, if the primary earner becomes unemployed compared to when the primary earner remains employed, confirming the existence of the added worker effect in our sample.[3] This result is in line with Guner, Kulikova, and Valladares-Esteban (2020), who find an overall AWE of 6.89 percentage points with CPS data spanning from 1976 to 2018 for couples between 25 and 54 years.

Zooming in on the precise margin of adjustment, we find that the conditional probability of the spouse transitioning directly into employment increases by 1.98 percentage points, whereas the conditional probability of the spouse transitioning into unemployment increases by 3.92 points. Thus, around two thirds of the overall AWE arise from individuals transitioning into unemployment, highlighting the importance of explicitly distinguishing between unemployed and non-participating individuals. Some couples may wish to leverage spousal labor supply as an insurance margin against job loss but labor market frictions (or the lack of appropriate job offers) may prevent them from doing so. If we only considered transitions from non-employment into employment, we would hence largely understate spousal labor supply adjustments in response to the job loss of the primary earner.

To further investigate the added worker effect, Table 2.2 splits primary earners by the reason for why they became unemployed. In particular, we distinguish between laid-off workers (who face a high chance of being recalled), job losers, workers whose temporary contracts ended, and voluntarily job leavers. Table 2.2, which splits the EU transition of the primary earner by reason for entering

[3] In this paper we focus on the transitions of out of the labor force spouses conditional on the labor market transitions of primary earners. In the appendix, Tables B.1 and B.2 we also report the conditional transition probabilities of unemployed and employed spouses, respectively. There is a slightly higher likelihood that unemployed spouses transition to employment or stay unemployed rather than leave the labor force if the primary earner loses the job compared to the primary earner staying employed. However, evidence for insurance through spousal labor supply is strongest when considering out of the labor force spouses, which we focus on.

Table 2.2 AWE by reasons of Unemployment for Household Head

	EE	EU (by reasons for U)			
		Layoff	Job Loser	Temp. Job ended	Job Leaver
NE	6.03%	6.13%	8.81%	7.56%	10.47%
NU	1.63%	3.51%	6.66%	6.59%	7.68%
NN	92.34%	90.35%	84.53%	85.85%	81.86 %

Notes: This table shows the added worker effect (as defined in the main text) by reason for the EU transition of the primary earner.

unemployment, shows that our finding is not solely driven by household members voluntarily quitting (column *Job Leavers*, especially with spouse NE) upon employment of their partner. The effect for those exogenously separated (*Job Losers*) is of similar magnitude, with a slightly decreased AWE for households in which the head's job loss can be seen as expected (*Temp. Job ended*) or as temporary in nature (*Layoff*).

While in the main text we focus on couples where one spouse is employed and the other is out of the labor force, in Appendix B.1 we include similar tables for couples that start as both employed or with one employed and one unemployed member. We can also see in these transition matrices that unemployed spouses are slightly more likely to enter employment or keep looking for jobs rather than dropping out of the labor force if the primary earner moves from employment to unemployment compared to when the primary earner stays employed. However, the main pattern that emerges from these two transition matrices is that couples often make joint transitions: The likelihood of a spouse dropping out of the labor force is drastically increased when the primary earner also transitions from employment or unemployment to out of the labor force.

The Added Worker Effect by Age

To analyze the life cycle dimension of the added worker effect, we split our sample into four age brackets and construct joint labor market transitions for each group in the same manner as above. Table 2.3 displays the results. We find a strong age-dependency in the strength of the AWE: For the youngest group (25 to 35 years), the likelihood that the spouse enters the labor force upon the job loss of the primary earner increases by 7.53 percentage points, for the young middle aged (36 to 45 years) it increases by 7.10 points, for the older middle aged (46 to 55 years) by 5.00 points, and eventually it only slightly increases by 1.29 points for the oldest group (56 to 65 years). Thus, spousal labor supply adjustments of the youngest age group are more than five times larger than for the oldest age group.

Table 2.3 Joint Labor Market Transitions by Age

	Primary earner transition		
	EE	EU	EN
Age Spouse 25-35:			
Cond. prob. of spousal NE transition	6.66%	9.30%	26.93%
Cond. prob. of spousal NU transition	2.00%	6.89%	2.02%
Cond. prob. of spousal NN transition	91.34%	83.81%	71.05%
Age Spouse 36-45:			
Cond. prob. of spousal NE transition	6.73%	9.32%	26.69%
Cond. prob. of spousal NU transition	1.86%	6.37%	2.00%
Cond. prob. of spousal NN transition	91.41%	84.31%	71.30%
Age Spouse 46-55:			
Cond. prob. of spousal NE transition	6.13%	7.96%	16.62%
Cond. prob. of spousal NU transition	1.62%	4.79%	1.72%
Cond. prob. of spousal NN transition	92.25%	87.25%	81.66%
Age Spouse 56-65:			
Cond. prob. of spousal NE transition	4.29%	3.73%	8.69%
Cond. prob. of spousal NU transition	0.90%	2.75%	0.56%
Cond. prob. of spousal NN transition	94.81%	93.52%	90.76%

Notes: This table shows the probability of a spousal transition from out of the labor force conditional on primary earner transitions by age group.

More specifically, for the young, we find behavioral responses both from non-participating directly into employment (2.64 percentage points) as well as into unemployment (4.89 percentage points). Thus, the relative share of young individuals transitioning directly into employment is slightly larger than for the entire sample. However, for the oldest age group, we only find small behavioral responses into unemployment (1.85 percentage points) and no response directly into employment (-0.56 points).

2.2.3 Dynamic Response

So far, we have focused on the contemporaneous spousal labor supply response, that is, the probability that a spouse enters the labor force in the *same month* as the head transitions into unemployment. This most likely understates the overall strength of the added worker effect since spousal labor supply responses may occur in prior months (anticipation effects) or some months delayed. In fact, Ellieroth (2019) documents spousal insurance not only in response to actual job loss of the primary earner but also in anticipation of such event, a phenomenon that she names “precautionary labor supply”. To analyze the strength of both anticipation and lagged responses, we run the following linear regression

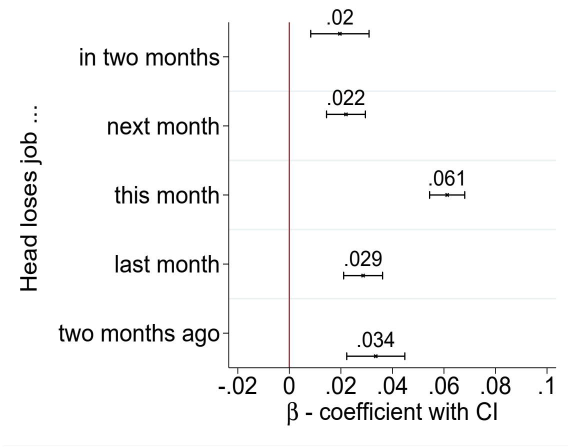


Figure 2.1 Δ Pr(Spouse enters LF) this month

Notes: Figure 2.1 shows the change in probability that a non-participating spouse enters the labor force (either as unemployed or as employed) if the household head loses/lost the job in two months, next month, this month, last month or two months ago, respectively, relative to the baseline in which the household head remains employed. The sample includes couples in which one spouse is working and one spouse is out of the labor force between age 25 and 65 from the Current Population Survey (CPS), waves 1994 until 2020. The regression producing the coefficients is Equation 2.1.

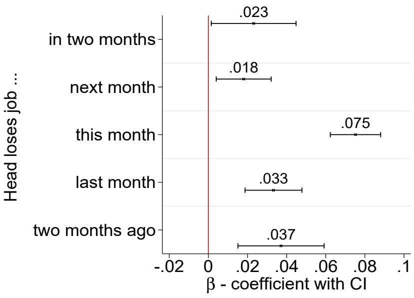
specification:

$$\Delta LFS_{it}^{sp} = \alpha_j + \beta_j \Delta ES_{it+j}^{h} + \gamma_j X_{it} + \varepsilon_{jit}, \tag{2.1}$$

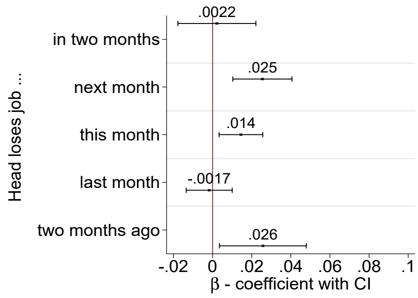
where ΔLFS_{it}^{sp} is a dummy that takes the value 1 if the non-participating spouse of couple i transitions either into employment or into unemployment between month $t-1$ and t, and 0 if she or he remains out of the labor force. Similarly, ΔES_{it}^{h} is defined as a dummy taking the value 1 if the primary earner transitions from employment into unemployment whereas it is 0 if the head stays in employment. X_{it} further controls for month fixed-effects, year fixed-effects, state fixed-effects, sex, race, education, children as well as the quarterly unemployment rate in the couple's state of residence.

Our coefficient of interest is β_j, indicating the likelihood that the spouse enters the labor force in month t if the household head transitions into unemployment in month $t+j$ versus when he or she remains employed (i.e. the strength of the AWE in month $t+j$). We conduct the analysis for $j = \{-2, -1, 0, 1, 2\}$. In the CPS, we observe the same couple for at most four consecutive months and hence a maximum of three consecutive labor market transitions, preventing us from considering more distant leads and lags. Figure 2.1 reports the results for the entire sample, whereas Figure 2.2 splits the observations by age.

In line with section 2.2.2, Figure 2.1 confirms the overall strength of the AWE of around 6.1 percentage points in the contemporaneous month. Moreover, this effect is statistically significantly different



(a) Age 25 to 35



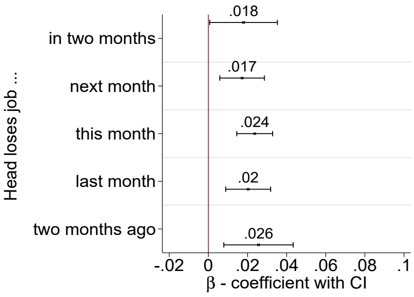
(b) Age 56 to 65

Figure 2.2 Δ Pr(Spouse enters LF) this month

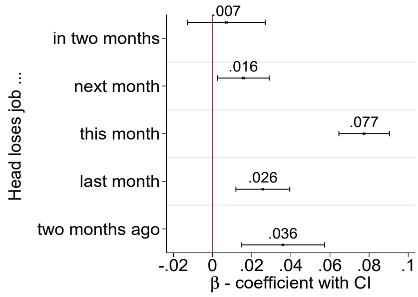
Notes: Figure 2.2 shows the change in probability that a non-participating spouse enters the labor force (either as unemployed or as employed) if the household head loses/lost the job in two months, next month, this month, last month or two months ago, respectively, relative to the baseline in which the household head remains employed. The sample includes couples in which one spouse is working and one spouse is out of the labor force between age 25 and 35 (Figure 2.2a) and between age 56 and 65 (Figure 2.2b) from the Current Population Survey (CPS), waves 1994 until 2020. Age refers to the non-participating spouse. The regression producing the coefficients is Equation 2.1.

from zero. In addition to the contemporaneous effect, we find strong support of both anticipation and lagged effects, albeit of lower magnitude. Overall, our results indicate that spousal labor supply responses in the months preceding and in the months after the primary earner's job loss are around half as strong as the direct response. When splitting the sample by age (Figure 2.2), we find that the contemporaneous effect is statistically significant for all age groups, however it is around five times stronger for the young than for the old. Moreover, young households display both lagged responses as well as anticipation effects, whereas we cannot confirm any clear pattern of those among households between 56 and 65 years. We relegate the results for the two middle age groups to Figure B.1 in the appendix.

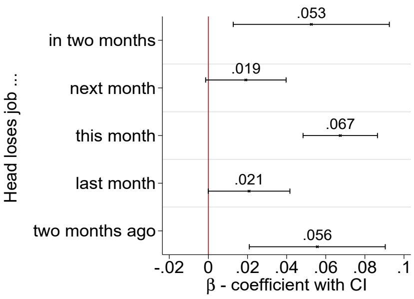
Lastly, in Figure 2.3, we again split the sample by reasons for unemployment of the primary earner (as in Table 2.2). Generally, the figure confirms that the probability that a non-participating spouse enters the labor force increases most if the EU transition of the primary earner is due to a quit or job loss, and less so in case of a layoff when there is a chance of being recalled. Interestingly, for spouses of household heads who voluntarily leave their job the effect two months ahead and the two month lagged effect are smaller, while the effect in the month before and after the primary earner transition is larger. This finding can be taken as indication that these labor market transitions are coordinated choices within a short time span.



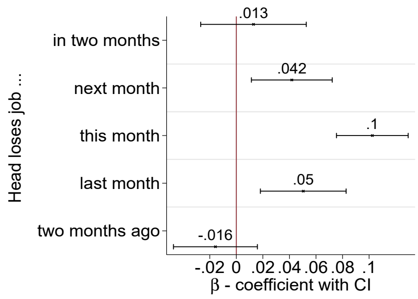
(a) Job Loser – On Layoff



(b) Job Loser



(c) Temporary Job Ended



(d) Job Leaver

Figure 2.3 Δ Pr(Spouse enters LF) this month

Notes: Figure 2.3 shows the change in probability that a non-participating spouse enters the labor force (either as unemployed or as employed) if the household head loses/lost the job in two months, next month, this month, last month or two months ago, respectively, relative to the baseline in which the household head remains employed; split by reasons for unemployment of the household head. Specifically, Figure 2.3a shows the results if the household head is on layoff, Figure 2.3b if the household head lost his job, Figure 2.3c if a temporary job ended and Figure 2.3d if the head voluntarily quit his or her job. The sample includes couples in which one spouse is working and one spouse is out of the labor force between age 25 and 65 from the Current Population Survey (CPS), waves 1994 until 2020. The regression producing the coefficients is Equation 2.1.

2.2.4 Robustness

In this section, we explore further channels that could result in the observed age-dependency in the added worker effect without relating to life cycle heterogeneity in the insurance value of the AWE itself and neither to other insurance margins that differ by age.[4] All corresponding tables are listed in Appendix B.1.

Education. If educational attainment differs by age and at the same time affects spousal labor supply responses, the stronger AWE for younger couples may simply arise from differences in education levels between old and young couples.[5] Indeed, Table B.3 confirms that the AWE is larger for spouses

[4]Some of these variables are also included as controls in the regressions. We still address the economically most important ones explicitly in this section.

[5]Generally, heterogeneity in education levels by age is low: around 45% of spouses among the youngest age group have a college degree, whereas around 40% of spouses among the oldest age group do.

with a college degree. However, when splitting the sample by age and education (Panel III to VI in Table B.3), the decreasing magnitude of the AWE over the life cycle holds both among spouses with a college degree and among those without a college degree.

Cohort Effects. If preferences for labor supply or within household insurance differ by cohorts, any age-dependency in the added worker effect between old and young couples may be driven by these underlying preference shifts. Female labor force participation increased substantially between the 1960s and the 1990s. Hence, entering the labor force upon the head's job loss may be easier for young couples if deviations from the traditional family model are societally more accepted. We address this concern in two ways. First, we split our sample by gender and age. Male labor force participation changed to a much lesser extent than that of women. Hence, if we can replicate the age-dependency in the AWE for couples in which the non-participating spouse is a man, possible cohort effects are less concerning. Table B.4 (Panels I and II) shows the results of this exercise. Although we find that the overall probability of the spouse joining the labor force is higher when the non-participating household member is a man, we do not find significant changes in the strength of the AWE (i.e. in the increased likelihood that the spouse enters the labor force when the household head becomes unemployed, compared to when the head remains employed). Focusing only on male non-participating spouses, young households still show a stronger AWE than older couples. We take this as suggestive evidence that our results are not driven by changing patterns of female labor force participation.

Of course, couples for which a man is non-participating could be a particular selection whose preferences differ from those of the remaining population. Therefore, we extract one cohort and repeat the empirical exercise on this restricted sample. In particular, we focus on couples in which the non-participating spouse was born between 1960 and 1970. We choose this timespan to ensure sufficiently many observations both for the young and for the old age brackets. Table B.4 (Panel III and IV) reports the results. Again, we can confirm the decreasing magnitude of the AWE over the life cycle for this particular cohort, i.e. for the same cohort when young and when old.

Children. Young couples are more likely to have children living in their household, which arguably affects labor supply behavior and could therefore result in the observed differences of spousal labor supply insurance. On the one hand, couples with children might have stronger incentives to enter the labor force in response to the job loss of the primary earner because they have larger consumption commitments and stronger saving motives (e.g. saving for college). On the other hand, if household members specialize in childcare and paid work, the willingness of the spouse who specializes in childcare to enter the labor force might be low. To address this issue, Table B.5 reports the AWE for couples below age 40 (to avoid picking up age-effects) with and without children as well as for couples below age 40 with and without children under age five (who require the most childcare). We do not find any (significant) differences in the overall strength of the AWE between couples with and without children across both specifications. As expected, out of the labor force spouses in couples without any children have a higher baseline probability of entering the labor force, independently of the labor market transition of the primary earner. However, we do not find any differential effect of the primary earner transitioning to unemployment on the probability to join the labor force between spouses in couples with and without children.

Reasons for Non-Participation. Individuals do not participate in the labor force for a variety of reasons that are age-dependent. At the same time, the reason for being out of the labor force can affect the strength of the added worker effect. For example, if the non-participating spouse is retired, transitioning back into the labor force has a much smaller insurance value because of pension payments. Similarly, if the non-participating spouse dropped out because of bad health, she or he might simply not be able to start working if the primary earner becomes unemployed. Arguably, both retirement and health related non-participation are more prevalent among the old. Therefore, Table B.6 repeats the empirical analysis excluding retired spouses (Panels I and II), disabled or ill spouses (Panels III and IV), as well as excluding both retired and disabled/ill spouses (Panels V and VI). Unsurprisingly, these restrictions do not impact our baseline results for the young age group in any way. However, we also do not find any significant impact on the strength of the AWE among the old. If anything, spouses are more likely to join the labor force in general when excluding retirees, however, the increase in the likelihood of entering (un)employment in response to the primary earner's job loss is not larger (or smaller) when repeating the analysis on the three subsamples.

Business Cycle. We investigate whether the results differ by the state of the business cycle, as much of the literature on joint search focuses on the business cycle (e.g. Mankart and Oikonomou (2016a) and Birinci (2019)). In Table B.7 we split the sample by NBER recessions and expansions. The state of the business cycle might matter for the added worker effect in several ways. On the one hand, if a primary earner loses a job in a recession, it might be harder to find a job again, so that insurance through spousal labor supply could be more important. On the other hand, it could also be harder for an out of the labor force spouse to find a job and provide this insurance. We do not, however, find large differences in the AWE across young and old for different states of the business cycle.

Income. A deficiency of the CPS for our analysis is that we do not observe asset holdings of households, which are another key insurance margin available to them. We have, however, some information on total income of a couple over the past year. This may proxy for the ability of households to build up savings, but is also correlated with other characteristics such as education. We split couples into income terciles and compute transition matrices for these different income groups. They are reported in Table B.8. Pooling all age groups we observe a sizeable AWE for low and high income groups. For the old, the added worker effect is relatively weak for both low and high income groups. When only considering the young, the AWE is smaller for the high income group than for the low income group. This may reflect that within the high income group the primary earner may have a higher chance of being reemployed or that the high income group has larger savings. Both these channels will be present in the our quantitative theory, to which we turn next.[6]

2.3 Model

The empirical evidence presented so far suggests that there is a significant age-dependency in the added worker effect: Spousal labour supply is a more important insurance margin for young than for old couples. We now build a life cycle search model with two-member households in order to better understand why the added worker effect is more prevalent among the young.

2.3.1 Environment

The economy is populated by two-member households. We assume that both members have the same age. Households live for T periods, after which they die deterministically. Households retire jointly after a working life of T_W periods, so that retirement lasts $T - T_W$ periods.

During working life an individual can be in one of four labor market states. An individual can be employed (E), in which case the agent receives a wage payment. If the individual does not have a job, there are three other labor market states: First, an agent may be unemployed and receive benefits (U). Second, the agent can be unemployed without receiving benefits (S). In both these states, the agent exerts costly search effort in order to increase the probability of finding a job. Third, an agent may choose to not exert this costly search effort. In that case, the agent is considered to be out of the labor

[6]In ongoing work, we extend our empirical analysis using data from the Survey of Income and Program Participation (SIPP). We started with the CPS as it is the main source for monthly labor market statistics in the United States. The SIPP, however, has the advantage that we can observe households' asset holdings.

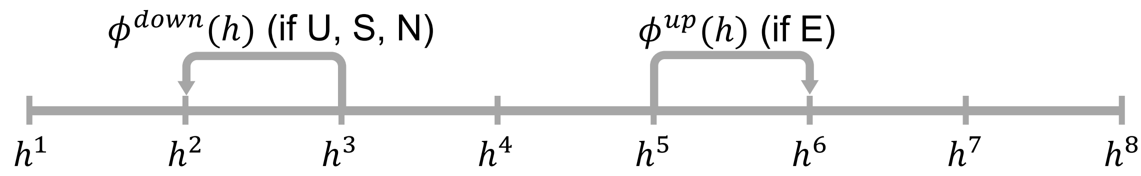


Figure 2.4 Human Capital Transitions

Notes: Figure 2.4 illustrates human capital transitions in the model.

force (N). Individuals who are not actively searching can never receive unemployment benefits. Given these four individual labor market states, there are 16 combined labor market states for a two-member household: $jk \in \mathscr{J} = \{E,U,S,N\} \times \{E,U,S,N\}$.

Each household member is endowed with a level of human capital, which evolves stochastically depending on the agent's employment status and current human capital level. If an individual member is employed, the human capital will go up by one unit with probability $\phi^{up}(h)$. For non-employed agents, human capital drops by one unit with probability $\phi^{down}(h)$. This process is illustrated in Figure 2.4.

While employed, an individual is also characterized by match quality z, which evolves according to a first-order Markov process. The match quality and the human capital level jointly determine the wage an individual receives. Non-employed individuals do not have a match quality, but draw one upon finding a new job.

Individual labor market transitions are illustrated in Figure 2.5. An employed agent can receive an exogenous separation shock with probability $\delta(h)$, which depends on the level of human capital. If such a separation shocks occurs, the agent transitions to unemployment and receives unemployment benefits. Note that in case of a separation shock an agent can choose to immediately leave the labor force instead of becoming unemployed and receiving benefits. This can be beneficial because no costly search effort is exerted while out of the labor force. If there is no separation shock, the individual can choose between staying employed and quitting. If she chooses to quit, she can either become unemployed without receiving benefits or leave the labor force entirely.

An unemployed agent who receives benefits can transition to all other labor market states. First, she receives a job offer with probability $\lambda^U(x_i)$ and transitions to employment if she chooses to accept the offer. The arrival rates with which non-employed agents receive job offers are endogenously determined as the solution to an optimal vacancy posting problem of firms (see below) and for household member i depend on state $x_i = \{h_i, h_{-i}, z_{-i}, a', jk\}$. An agent can choose to reject an offer and might do so if the initial match quality draw is low. In that case, it may be preferable to wait for a new offer with

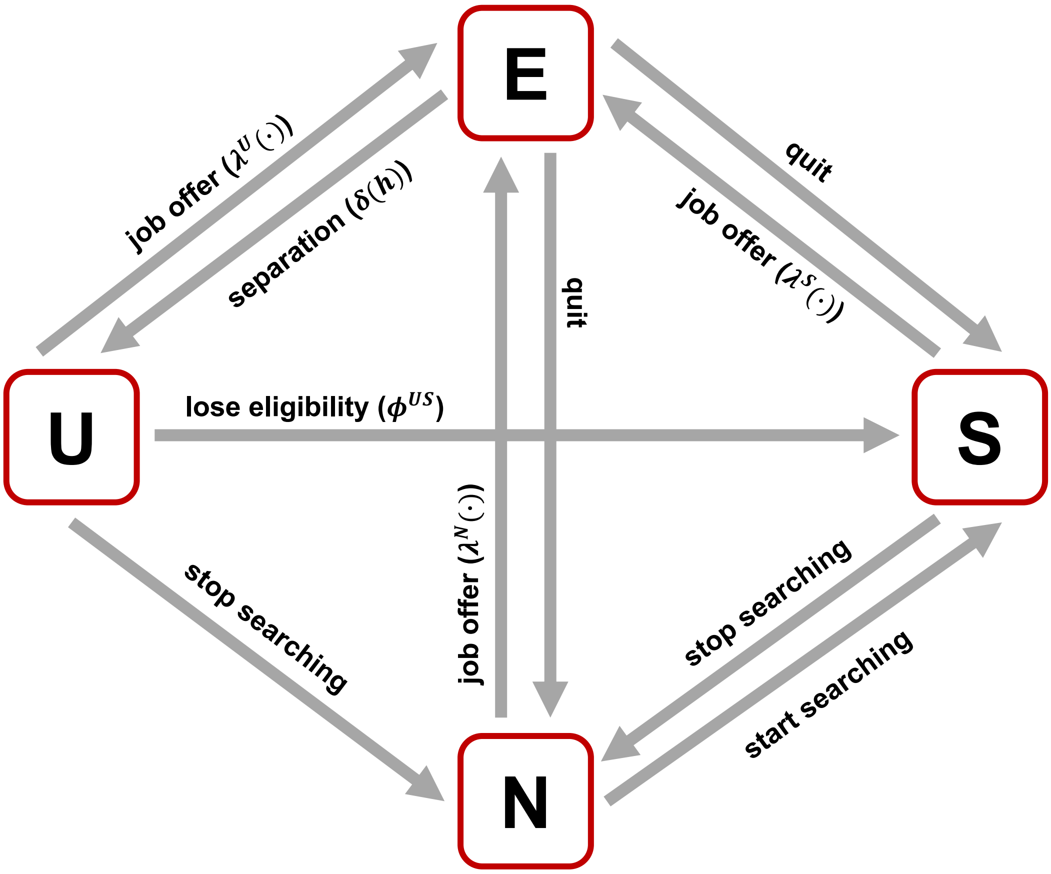


Figure 2.5 Labor Market Transitions in the Model

Notes: Figure 2.5 illustrates possible labor market transitions in the model. $x_i = \{h_i, h_{-i}, z_{-i}, a', jk\}$ is the relevant state for the arrival rate of household member i.

a potentially better match quality draw. Second, an unemployed worker who receives benefits can stochastically lose benefit eligibility with probability ϕ^{US}. This captures that unemployment benefits run out after a certain time period. Third, she can choose to stop searching and leave the labor force. Similarly, an unemployed worker without benefits receives job offers with probability $\lambda^S(x_i)$ and can quit the labor force.

Finally, out of the labor force agents receive job offers with probability $\lambda^N(x_i)$, even though they do not exert active search effort. This assumption is necessary to capture the empirical observation that individuals directly transition from out of the labor force into employment. Moreover, non-participating agents can rejoin the labor force as unemployed without benefits.

While each household member has an individual labor market state, human capital level, and match quality shock when employed, households jointly have access to a risk-free bond. They can save in this bond at the exogenous interest rate r. Borrowing is not allowed.

Table 2.4 Labor Supply Choice Sets

Benefit Eligibility	**Job (Offer)** Both	Member 1	Member 2	None
Both	$\mathscr{J}^{EE}_{UU} = \{E,U,N\} \times\{E,U,N\}$	$\mathscr{J}^{EX}_{UU} = \{E,U,N\} \times\{U,N\}$	$\mathscr{J}^{XE}_{UU} = \{U,N\} \times\{E,U,N\}$	$\mathscr{J}^{XX}_{UU} = \{U,N\} \times\{U,N\}$
Member 1	$\mathscr{J}^{EE}_{UX} = \{E,U,N\} \times\{E,S,N\}$	$\mathscr{J}^{EX}_{UX} = \{E,U,N\} \times\{S,N\}$	$\mathscr{J}^{XE}_{UX} = \{U,N\} \times\{E,S,N\}$	$\mathscr{J}^{XX}_{UX} = \{U,N\} \times\{S,N\}$
Member 2	$\mathscr{J}^{EE}_{XU} = \{E,S,N\} \times\{E,U,N\}$	$\mathscr{J}^{EX}_{XU} = \{E,S,N\} \times\{U,N\}$	$\mathscr{J}^{XE}_{XU} = \{S,N\} \times\{E,U,N\}$	$\mathscr{J}^{XX}_{XU} = \{S,N\} \times\{U,N\}$
None	$\mathscr{J}^{EE}_{XX} = \{E,S,N\} \times\{E,S,N\}$	$\mathscr{J}^{EX}_{XX} = \{E,S,N\} \times\{S,N\}$	$\mathscr{J}^{XE}_{XX} = \{S,N\} \times\{E,S,N\}$	$\mathscr{J}^{XX}_{XX} = \{S,N\} \times\{S,N\}$

Notes: This table shows the labor supply choice sets of households.

2.3.2 Household Search Problem

Timing in the model is as follows: In each period, households first receive their labor income (wages or unemployment benefits) as well as their asset income from investing in the risk-free bond. Given their budget constraint, households then make a consumption-savings choice. Afterwards, first separation shocks, job offers as well as potential losses of benefit eligibility are realized for both household members in parallel. Afterwards, match quality shocks and human capital transitions are revealed. Finally, households choose their joint future labor market state from the feasible subset of $\mathscr{J}$, which is determined by their previous labor market state and job offers, separations, and benefit eligibility losses.

Table 2.4 summarizes all possible combinations of job opportunities and unemployment benefit eligibility of the two household members along with the associated choice sets over joint labor market states. The superscripts to $\mathscr{J}$ indicate whether the household members have the opportunity to be employed. An employment opportunity arises either because an agent was employed in the previous period and did not receive a separation shock or because an agent received a job offer while non-employed. If both members have the opportunity to be employed, the superscript is EE. In contrast, X indicates that a member cannot be employed. Hence, EX and XE are the cases where only one member has a job opportunity, whereas XX indicates that neither household member can be employed in the following period.

The logic for the subscripts is similar. However, they refer to unemployment benefit eligibility of the individual household member. Again, U indicates eligibility, while X refers to non-eligibility.

We are now in the position to formally state the household search problem. The value function of a household of age t in joint labor market state jk is

$$V_t^{jk}(z,h,a) = \max_{a'} u(c^{jk}(z,h,a,a')) + \psi_t^{jk} + \beta \Theta_{t+1}^{jk}(z,h,a'), \tag{2.2}$$

where the additional state variables are the match quality shocks of both household members ($z = (z_1, z_2)$), their human capital levels ($h = (h_1, h_2)$), and joint asset holdings a. Households value consumption c according to the utility function $u(c)$. Consumption is pooled within the household. Additionally, instantaneous utility is affected by ψ which is allowed to depend on the labor market state and age. It captures disutility from search and the utility of staying at home. Households discount their continuation value Θ, which is described in detail below, with discount factor β.

Households choose assets for the next period subject to their budget constraint

$$c^{jk}(z,h,a,a') = \underbrace{\mathbb{I}_{j=E} w(z_1,h_1) + \mathbb{I}_{k=E} w(z_2,h_2)}_{\text{labor income}} + \underbrace{\mathbb{I}_{j=U} \bar{b} + \mathbb{I}_{k=U} \bar{b}}_{\text{unemployment benefits}} - \underbrace{(a' - (1+r)a)}_{\text{net savings}}. \tag{2.3}$$

Depending on their employment status households receive wage and benefit income. In addition to this, a household can use its assets and interest income to finance consumption and new purchases of the risk-free bond.

To write the continuation utility for one labor market state explicitly, we consider a household with two employed members today. Since both members are employed, the relevant state variables are two match quality shocks and two human capital levels. In addition, the continuation utility depends on the asset choice.

We express the continuation value in two steps. First, we take expectations over separation shocks and and the resulting choice sets for future labor market states:

$$\begin{aligned}
&\Theta_{t+1}^{EE}(z_1,z_2,h_1,h_2,a') = \\
&\quad (1-\delta(h_1))(1-\delta(h_2))\, \widetilde{V}_{t+1}(z_1,z_2,h_1,h_2,a',\mathscr{J}_{XX}^{EE}) \\
&\quad +\delta(h_1)(1-\delta(h_2))\, \widetilde{V}_{t+1}(z_1,z_2,h_1,h_2,a',\mathscr{J}_{UX}^{XE}) \\
&\quad +(1-\delta(h_1))\delta(h_2)\, \widetilde{V}_{t+1}(z_1,z_2,h_1,h_2,a',\mathscr{J}_{XU}^{EX}) \\
&\quad +\delta(h_1)\delta(h_2)\, \widetilde{V}_{t+1}(z_1,z_2,h_1,h_2,a',\mathscr{J}_{UU}^{XX}).
\end{aligned} \tag{2.4}$$

If neither member is exogenously separated (first line), both household members have the opportunity to work, but neither of them is eligible for benefits if he or she chooses to voluntarily quit. Hence, the feasible set of labor market states is denoted by $\mathscr{J}_{XX}^{EE}$. Lines 2 and 3 deal with the cases in which one member is exogenously separated whereas the last line considers the case in which both members receive the separation shock. In these instances, the exogenously separated member is eligible for benefits but cannot be employed in the next period.

In a second step, we consider transitions for match quality z and human capital h as well as the household's discrete choice over feasible future labor market states:

$$
\begin{aligned}
&\widetilde{V}_{t+1}(z_1, z_2, h_1, h_2, a', \mathscr{J}^{OP}_{QR}) = \\
&\quad \phi^{up}(h_1)\phi^{up}(h_2)\, \mathbb{E}_{z_1'|z_1}\mathbb{E}_{z_2'|z_2}\, \mathbb{E}_{\varepsilon} \max_{\widehat{jk}\in\mathscr{J}^{OP}_{QR}} \left\{ V^{\widehat{jk}}_{t+1}(z_1', z_2', h_1+1, h_2+1, a') + \sigma\varepsilon^{\widehat{jk}} \right\} \\
&+\phi^{up}(h_1)(1-\phi^{up}(h_2))\, \mathbb{E}_{z_1'|z_1}\mathbb{E}_{z_2'|z_2}\, \mathbb{E}_{\varepsilon} \max_{\widehat{jk}\in\mathscr{J}^{OP}_{QR}} \left\{ V^{\widehat{jk}}_{t+1}(z_1', z_2', h_1+1, h_2, a') + \sigma\varepsilon^{\widehat{jk}} \right\} \\
&+(1-\phi^{up}(h_1))\phi^{up}(h_2)\, \mathbb{E}_{z_1'|z_1}\mathbb{E}_{z_2'|z_2}\, \mathbb{E}_{\varepsilon} \max_{\widehat{jk}\in\mathscr{J}^{OP}_{QR}} \left\{ V^{\widehat{jk}}_{t+1}(z_1', z_2', h_1, h_2+1, a') + \sigma\varepsilon^{\widehat{jk}} \right\} \\
&+(1-\phi^{up}(h_1))(1-\phi^{up}(h_2))\, \mathbb{E}_{z_1'|z_1}\mathbb{E}_{z_2'|z_2}\, \mathbb{E}_{\varepsilon} \max_{\widehat{jk}\in\mathscr{J}^{OP}_{QR}} \left\{ V^{\widehat{jk}}_{t+1}(z_1', z_2', h_1, h_2, a') + \sigma\varepsilon^{\widehat{jk}} \right\}
\end{aligned}
\tag{2.5}
$$

For employed individuals human capital can either remain constant or increase. Each line of equation 2.5 corresponds to one of the resulting four combinations of possible human capital transitions. Moreover, in each case, expectations are also taken with respect to match quality shocks.

The possible choices of future labor market states can be read off Table 2.4. $\varepsilon \in \mathbb{R}^{|\mathscr{J}^{OP}_{QR}|}$ is a vector of iid, Type-I extreme value (Gumbel) shocks with mean zero. We introduce these taste shocks for computational purposes, as they smooth out kinks and discontinuities in the policy functions that arise from the discrete choices over labor market states. We choose the variance of these taste shocks to be small enough such that they do not affect the solution to the problem in an economically meaningful way.

While we outline here the continuation value for a household with two employed members today, the problem for all other current joint labor market states evolves in a very similar manner: In equation 2.4, instead of separation shocks expectations are formed over job offer arrivals and potential losses of benefit eligibility for non-employed members. Equation 2.5 remains mostly unaffected except for initial draws of z out of non-employment, which stem from an initial distribution and are unaffected by past realizations of z.

2.3.3 Vacancy Posting and Endogenous Arrival Rates

To determine the job arrival rates of households endogenously we look at the optimal vacancy posting problem of single-job firms. We assume free entry of firms and a cost κ of posting a vacancy. A vacancy lasts for one period and if not filled can be renewed by paying κ again.

A match with quality z between a firm and a worker with human capital h produces per period output $y(z,h)$, of which the worker receives a constant share χ as a wage $w(z,h) = \chi y(z,h)$, yielding firms' per period profit of such match as $\pi(z,h) = (1-\chi)y(z,h)$.

The expected future value to a firm of a match with a worker i from a household with current state $x_i = (t, z_i, z_{-i}, h_i, h_{-i}, a, jk)$ and asset choice for next period a' , given that the household can choose the joint future labor market state from set $\mathscr{J}_{QR}^{OP}$, is defined as

$$
\begin{aligned}
&EJ_{t+1}^{jk}(z_1, z_2, h_1, h_2, a', \mathscr{J}_{QR}^{OP}) = \\
&\qquad \mathbb{E}_{h_1'|h_1}\mathbb{E}_{h_2'|h_2}\mathbb{E}_{z_1'|z_1}\mathbb{E}_{z_2'|z_2}\mathbb{E}_{\hat{j}k\in\mathscr{J}_{QR}^{OP}}\mathbb{I}_{\hat{j}=E|x'}J_{t+1}^{\hat{j}k}(z_1', z_2', h_1', h_2', a')
\end{aligned}
\tag{2.6}
$$

where $\mathbb{E}_{\hat{j}k\in\mathscr{J}_{QR}^{OP}}\mathbb{I}_{\hat{j}=E|x'}$ is firms' expectation of the household's joint labor market choice and an indicator of whether for each joint state member i stays with the firm, i.e. is firms' expectation over endogenous acceptances and quits. The contemporaneous value to the firm is then given by

$$
J_t^{jk}(z_1, z_2, h_1, h_2, a) = \pi(z_1, h_1) + \frac{1}{1+r}(1-\delta(h_1))\mathbb{E}_{P,R}EJ_{t+1}^{jk}(z_1, z_2, h_1, h_2, a', \mathscr{J}_{XR}^{EP}), \tag{2.7}
$$

where $\mathbb{E}_{P,R}$ is a firm's expectation over job loss, job finding, and eligibility transitions of the spouse and $a' = a(t, z_1, z_2, h_1, h_2, a, jk)$ is the household's asset choice.

We discuss the determination of endogenous arrival rates using the example of a household with both members unemployed but not eligible for benefits, i.e. a household with initial labor market state SS. Define member i's arrival rate as

$$
\lambda_t(h_i, h_{-i}, a, jk) = \lambda_S p(\theta_t(h_i, h_{-i}, a, jk)) \tag{2.8}
$$

with arrival rate $p(\theta) = m(1,\theta)$ and corresponding vacancy filling rate $q(\theta) = m(\frac{1}{\theta}, 1)$, where $m(U,V)$ is the standard Cobb-Douglas matching function, with market tightness θ denoting the ratio of vacancies over searchers in any given submarket. Hence $p(\theta) = \theta^{1-\alpha}$, $q(\theta) = \theta^{-\alpha}$, and $p(\theta) = \theta q(\theta)$. The λ_S is an exogenous shifter that only depends on the previous labor market state and reflects the consequences of differences in search effort between unemployed (U or S) and out of the labor force (N). This is necessary to impose as – conditional on the remaining states of the household – firms will not differentiate whether they hire a worker out of unemployment or out of the labor force.

Free entry imposes that the expected value of a vacancy (probability of filling times the value if filled) has to equal the cost of posting κ. This condition determines relevant market tightness $\theta_t(h_i, h_{-i}, a, jk)$. The free entry condition needs to satisfy

$$\kappa = q(\theta_t(h_i, h_{-i}, a, jk))\mathbb{E}_P EJ_{t+1}^{jk}(z_1, z_2, h_1, h_2, a', \mathscr{I}_{XX}^{EP}). \tag{2.9}$$

Here $\mathbb{E}_P$ captures expectations over the spouse's job finding and is an equation in the spouse's $\theta_t(h_{-i}, h_i, a, jk)$ as the spouse is also currently not employed. Hence, in all cases with currently two non-employed household members we have to solve a system of two non-linear equations in two unknowns.

With slight abuse of notation the two equations solving for two θs can be written as

$$\kappa = q(\theta_1)[\lambda(\theta_2)\underbrace{EJ_{t+1}^{SS}(h_1, h_2, a', \mathscr{I}_{XX}^{EE})}_{EJ_1^{EE}} + (1-\lambda(\theta_2))\underbrace{EJ_{t+1}^{SS}(h_1, h_2, a', \mathscr{I}_{XX}^{EX})}_{EJ_1^{EX}}], \tag{2.10}$$

$$\kappa = q(\theta_2)[\lambda(\theta_1)\underbrace{EJ_{t+1}^{SS}(h_2, h_1, a', \mathscr{I}_{XX}^{EE})}_{EJ_2^{EE}} + (1-\lambda(\theta_1))\underbrace{EJ_{t+1}^{SS}(h_2, h_1, a', \mathscr{I}_{XX}^{EX})}_{EJ_2^{EX}}]. \tag{2.11}$$

This yields

$$\theta_2 = \left[\frac{\kappa}{\lambda(\theta_1)EJ_2^{EE} + (1-\lambda(\theta_1))EJ_2^{EX}}\right]^{-\frac{1}{\alpha}} \tag{2.12}$$

and hence

$$\begin{aligned}\kappa = q(\theta_1)&\left[\lambda_S\left[\frac{\kappa}{\lambda(\theta_1)EJ_2^{EE} + (1-\lambda(\theta_1))EJ_2^{EX}}\right]^{\frac{\alpha-1}{\alpha}} EJ_1^{EE}\right.\\ &+\left.\left(1-\lambda_S\left[\frac{\kappa}{\lambda(\theta_1)EJ_2^{EE} + (1-\lambda(\theta_1))EJ_2^{EX}}\right]^{\frac{\alpha-1}{\alpha}}\right)EJ_1^{EX}\right],\end{aligned} \tag{2.13}$$

which is a non linear equation in one unknown and can be solved numerically.

The endogenous arrival rates can be derived in a similar fashion for other cases of original labor market states. The exogenous component of λ needs to be adjusted to reflect whether an agent is unemployed or out of the labor force. Solving for endogenous arrival rates gets substantially easier if one spouse has been previously employed since in this case we only have one θ and hence we only need to solve one equation with one unknown.

Given this setup, the job finding probabilities of an individual depend on all the state variables, including assets, age, and own human capital, but also the spouse's human capital, employment status, and potentially match quality. For age, this is exactly what we want to get out of this problem. As agents get older it may be harder for them to find new jobs, as firms are reluctant to hire them. This is captured in this setup because older agents have to retire at a certain age and therefore there is less time for for firms to recover the vacancy posting cost. In our calibration, this effect is strong close to retirement but relatively weak at young ages because for young individuals it is quite likely that the match is dissolved before retirement in any case.

It is also intuitive that arrival rates depend on an individual's human capital. It is potentially less appealing that we also condition on the spouse's state variables. It is necessary, however, because it influences the probabilities of an individual accepting a certain job and quitting later on. Having different submarkets and free entry in each active submarket simplifies computation drastically, as we do not need to know the distribution of individuals across states to solve for arrival rates.

This setup for determining age-dependent arrival rates in the labor market generally implies arrival rates decreasing in age, decreasing in assets because richer individuals are more likely to quit, increasing in human capital because the value of the match is higher and individuals are less likely to quit, increasing in match quality for the same reasons, and decreasing in a spouse's employment, human capital, and match quality because having a spouse earning high wages increases the quit probability and lowers the value of a match to the firm.

2.3.4 Numerical Implementation

In our setup, agents do not face risk during retirement. This assumption renders the household problem during retirement very simple. We solve the retirement problem using the endogenous grid method of Carroll (2006) to obtain a terminal condition for the household problem during working life.

The household problem during working life is high-dimensional because of the many combinations of labor market states and the fact that we have to keep track of match quality shocks and human capital for both members. Furthermore, given our focus on labor market transitions, the model has a monthly frequency. For computational efficiency, we therefore solve the household problem following Iskhakov, Jørgensen, Rust, and Schjerning (2017), who extend the endogenous grid-point method of Carroll (2006) to problems with discrete and continuous choices. Thus, their approach is well suited for our problem with a discrete choice over labor market states and a continuous asset choice.

The algorithm proceeds as follows: Within each period, given future value functions of both the household and firm, we begin by determining households' choices over future labor market states for each potential choice set. With this, we are able to solve firms' vacancy posting problem and determine endogenous arrival rates. Endogenous arrival rates given, we can solve households' consumption-savings problem as described above. In a final step, we update households' and firms' value functions making use of households' policy functions and again the endogenous arrival rates.

2.4 Calibration

We solve the model at a monthly frequency. This assumption is in line with the frequency at which we observe labor market transitions in the data and necessary because the U.S. labor market exhibits high rates of turnover. We assume that the period of working life is 40 years, corresponding to 480 months. The retirement period is another 120 months, i.e. 10 years.

2.4.1 Functional Form Assumptions

Households value consumption with a standard CRRA utility function

$$u(c) = \frac{c^{1-\gamma} - 1}{1 - \gamma}, \tag{2.14}$$

where γ is the coefficient of relative risk aversion. The second part of instantaneous utility that has to be parameterized is the parameter ψ_t^{jk}, which captures the disutility of search and utility of staying at home. Naturally, this parameter can differ depending on the joint labor market state reflecting disutility of work and search. Furthermore, we allow it to vary by age.[7]

Output is assumed to be the product of human capital and the match quality shock:

$$y(h,z) = hz. \tag{2.15}$$

Human capital is defined on an equidistant grid. The probabilities of moving to a higher (lower) human capital level when employed (non-employed) are given by the following processes:

$$\phi^{up}(i) = \bar{\phi}^{up} i^{\underline{\phi}^{up}} \tag{2.16}$$

$$\phi^{down}(i) = \bar{\phi}^{down} i^{\underline{\phi}^{down}}, \tag{2.17}$$

[7] In the current calibration, the disutility of search parameter is mostly constant across age. In fact, we make an exception only for one labor market state, as discussed below.

where i indicates the grid point rather than the level of human capital. This process is flexible enough to capture falling or rising probabilities of moving up or down the human capital ladder. The match quality shock while employed is assumed to follow an autoregressive process of order 1 in logs. We discretize the process using the method of Tauchen (1986).

Finally, we have to make an assumption on the arrival rates of job offers and separation rates in the labor market. We restrict $\lambda_S, \lambda_U, \lambda_N$ to be constant across age.[8] We allow the separation rate to vary with human capital according to a similar process as the probabilities of moving up or down the human capital ladder:

$$\delta(i) = \bar{\delta} i^{\underline{\delta}}. \tag{2.18}$$

2.4.2 Parameters and Moments

To compare the model to the data, we simulate the full life cycle of 160,000 households and compute model-implied moments of this simulation. We initialize the distribution of households across labor market states such that it is consistent with the data. We assume that all agents start with one of the lowest asset levels. For employed individuals, we draw the match quality shock from the stationary distribution of the match quality process. For human capital, even though this is mostly supposed to capture work experience in our model, we assume some heterogeneity in the initial distribution to obtain sufficient dispersion in incomes. Human capital levels are, however, concentrated on the lower rungs of the human capital ladder.

While in the model all parameters jointly determine all moments, we now discuss which parameters are most closely related to which moments. Table 2.5 summarizes the parameter values. We start by setting a number of parameters without solving the model. We exogenously fix the coefficient of relative risk aversion to two, a standard value in the literature. We also exogenously fix a monthly net interest rate of 0.17%, corresponding to an annual interest rate of roughly 2%. We also fix the probability of losing unemployment benefits $\phi^{US} = 1/6$, consistent with an average duration of benefit receipt of six months. Finally, we set the elasticity of the matching function α to 0.5, as in Petrongolo and Pissarides (2001), and the share of match output going to the worker χ to 0.7.

We target key moments of the U.S. labor market that are related to a large number of parameters. First, we target individual transition rates between labor market states. These are closely related to the parameters $\lambda_N, \lambda_S, \lambda_U$, the exogenous upper bounds on arrival rates depending on labor market states. We

[8] Even though the exogenous component of arrival rates is constant in age, the solution to firms' vacancy posting problem endogenously yields arrival rates falling in age t conditional on households' remaining states.

Table 2.5 Parameter Values

Parameter	Interpretation	Value
Demographics		
T	Length of life in months	600
T_W	Length of working life in months	480
Preferences		
β	Discount factor	0.9955
γ	Risk aversion	2.0000
$\psi^{EE}, \psi^{EU}, \psi^{UE}, \psi^{ES}, \psi^{SE}$	Disutility of work/search	0.0000
$\psi^{UU}, \psi^{SS}, \psi^{SU}, \psi^{US}$	Disutility of work/search	0.5000
$\psi^{UN}, \psi^{NU}, \psi^{SN}, \psi^{NS}$	Disutility of work/search	1.2000
ψ^{NN}	Disutility of work/search	2.6000
ψ^{EN}, ψ^{NE}	Disutility of work/search	$1.3+\frac{0.9-1.3}{1+e^{-0.05(t-100)}}$
Financial Assets		
r	Interest rate	0.0017
Labor Market		
$\bar{\delta}$	Level parameter separation rate	0.0200
$\underline{\delta}$	Curvature parameter separation rate	-0.5000
λ_U, λ_S	Probability of job offer for unemployed	0.4500
λ_N	Probability of job offer out of labor force	0.3000
Human Capital		
$\underline{h}$	Lower bound h	0.2000
$\bar{h}$	Upper bound h	0.8000
$\bar{\phi}^{up}$	Level parameter prob. h rise	0.0500
$\underline{\phi}^{up}$	Curvature parameter prob. h rise	-1.2000
$\bar{\phi}^{down}$	Level parameter prob. h fall	0.3316
$\underline{\phi}^{down}$	Curvature parameter prob. h fall	0.0000
Match Quality Shocks		
ρ_z	Persistence	0.9000
σ_z	Standard deviation	0.1000
Firms		
χ	Labor share of output	0.7000
κ	Cost of vacancy posting	8.0000
α	Matching elasticity	0.5000
Government		
b	Unemployment benefit	0.2500
ϕ^{US}	Probability of losing benefits	0.1667
p	Pension	0.2000
Gumbel shock		
σ_ε	Standard deviation of taste shock	0.1000

Notes: Table 2.5 summarizes the parameter values.

impose the restriction $\lambda_S = \lambda_U$, as these two states are supposed to only differ in whether an individual receives unemployment benefits or not. Individual transition rates are also closely related to the vacancy posting cost κ. The EU rate in particular pins down parameters of the job loss process. The model captures well the magnitude of the transitions between employment and unemployment. The model undershoots the magnitude of transitions between non-employment and employment/unemployment, as we will discuss in more detail in the next section when looking at the added worker effect in the model.

Another important set of labor market moments that we target is the distribution of households over joint labor market states for four ten-year age groups. Because the arrival rates are endogenously determined from the firm problem we treat the preference parameters ψ governing the disutility of work and search as free parameters to match joint labor market states by age. We keep all these parameters constant by age, except for $\psi^{EN} = \psi^{NE}$, which we assume to be decreasing with age. Specifically, these parameters start at a level of 1.30 at age 25 and decay logistically to a level of 90 with a half-life of 100 months. We need this because otherwise too many young households have both members employed. Economically, it makes sense that there is a higher utility of having one household member at home for the young age group because this is the age group that may have young children to take care of. As we do not model children explicitly, introducing age-dependency in ψ is a simple way of capturing this motive and helps us to match a high enough share of young households with one member employed and one member out of the labor force.

In addition to these labor market moments, we target life cycle profiles of income and assets. The level of the pension p and the discount factor β are mostly determined by the shape of the life cycle asset profile. Specifically, we target mean asset holdings for four age groups. An important question is which assets to consider in the data when constructing the moments to be matched. For insurance reasons, the relevant concept is liquid assets. In particular, because a model period is one month, it would be desirable to consider only assets that can be liquidated at a monthly frequency. However, given the life cycle dimension of our setup, retirement is an important driver of savings. Imposing too strict a requirement on asset liquidity would exclude much of households' retirement savings. Therefore, considering the trade-off between asset liquidity and retirement savings, we choose to target financial assets including retirement accounts net of debt. In addition, we include vehicle equity because it can be accessed very quickly. However, we exclude houses and mortgages because tapping into home equity is difficult for unemployed and might take longer, so it is not as useful for insurance purposes on a monthly frequency. Business equity is excluded for the same reason. We construct asset-related data moments from the Panel Study of Income Dynamics (PSID).

The parameters of the human capital process are chosen to match the income profile over the life cycle. In the data, these moments are also constructed from the PSID. The probability of moving up the human capital ladder is decreasing in the human capital level which is a way of achieving a concave income profile: When young, an agent moves up the human capital ladder quickly such that the wage increase is steeper. After a few steps on the human capital ladder, the likelihood of a further increase in human capital goes down quite significantly such that the income profile becomes flatter. The probability of losing human capital, by contrast, is constant across human capital levels. Human capital decay of non-employed allows us to capture the empirical observation that newly employed individuals have lower wages than long-time employed and that job losses lead to persistent wage losses (Davis and von Wachter, 2011; Jarosch, 2015; Kospentaris, 2021).

The parameters of the match quality shock process are chosen to match the variance in income levels by age group. Additionally, we have to pin down the distribution from which newly employed draw their match quality, which we set to the stationary distribution of the discretized Markov chain.

The only remaining parameters to be set are the level of the unemployment benefit and the variance of the taste shock. We assume the unemployment benefit to be constant and set its level to be roughly 50% of median income. For the taste shock, we set $\sigma_\varepsilon = 0.1$. Using 0.05 instead does not meaningfully impact the results.

2.4.3 Fit of Targeted Moments

In this section, we present the model fit for key targeted moments. First, Figure 2.6 shows the share of households in joint labor market states by age group in the model and in the data. To compare the model to the data, we pool all agents who are unemployed with and without benefits into one group, labeled U. In all age groups, the most common joint labor market state is that both members are employed. This share is, however, strongly decreasing in age, with around 65% of households being in that group among the two young groups and just 45% in the oldest age group. By contrast, the share of households where at least one member is out of the labor force is increasing over the life cycle. Among the youngest there are very few households with both members out of the labor force. Among the oldest, this share is almost 20%. Also, the share of households where one member is employed and one member is out of the labor force is slightly increasing in age. Overall, the model captures well the distribution of households over joint labor market states. It also captures that the share of two earner households is decreasing in age and the share of households with at least one member out of the labor force is increasing with age, though it somewhat understates the magnitude of these changes over the life cycle.

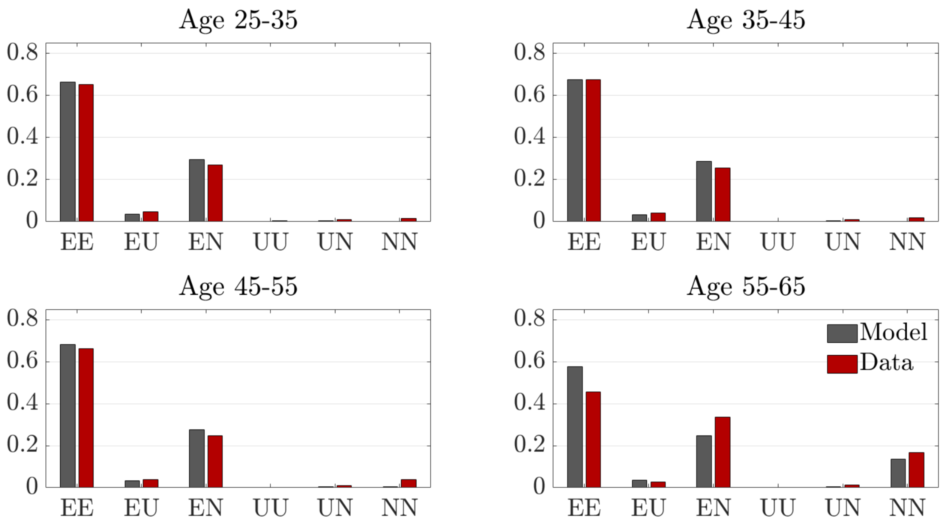


Figure 2.6 Joint Labor Market States of Couples (Model vs. Data)

Notes: Figure 2.6 shows the joint labor market states of couples in the model and in the data. For the model, U includes both unemployed receiving benefits and searchers who do not receive benefits. The data is from the CPS.

Table 2.6 Asset Levels

	Model	Data
All	10.4	11.8
Age 25-35	2.8	3.0
Age 36-45	4.9	7.0
Age 46-55	10.6	14.6
Age 55-65	23.3	24.1

Notes: Table 2.6 compares mean asset holdings by age group in the model and in the data. The data is from the PSID. In the data, assets include financial assets net of debt and vehicle equity. 1 unit corresponds to $10,000.

Additionally, the model matches the average asset holdings over the life cycle well, as shown in Table 2.6. Averaging over all age groups, we match the mean asset level of the population well. However, the model slightly underpredicts the mean asset holdings of the medium age groups. Still, the model is able to capture that average asset holdings are strongly increasing in age.

Finally, we consider the model fit for the mean income levels and the dispersion in income across age groups. Table 2.7 shows the comparison between data and model. Again, when averaging over all age groups, the model is close to the income level in the data but as of now undershoots the dispersion. Moreover, the model is able to replicate the increase in mean income for the age groups 25-35, 35-45, and 45-55. It fails, however, in generating a fall in income for the oldest group. This mismatch for the oldest age group arises from a very strong selection effect in the model who stays in the labor force.

Table 2.7 Income Levels and Dispersion

	Level		Standard deviation	
	Model	Data	Model	Data
All	0.3594	0.3424	0.1363	0.2374
Age 25-35	0.3296	0.3020	0.1172	0.2009
Age 36-45	0.3538	0.3572	0.1341	0.2456
Age 46-55	0.3752	0.3629	0.1429	0.2486
Age 56-65	0.3826	0.3400	0.1511	0.2466

Notes: Table 2.7 compares mean and standard deviation of labor income by age group in the model and in the data. The data is from the PSID. 1 unit corresponds to $10,000.

Many agents with relatively low human capital and/or match quality prefer to drop out of the labor force, which drives up the average income among the employed. In contrast, the model does replicate that income dispersion within age group is higher among the old than the young.

2.5 Results

In this section we first present the model implications for untargeted moments. Second, we show that our model can replicate the decreasing magnitude of the added worker effect over the life cycle. Third, we use the model to construct counterfactuals and analyze which channels are responsible for the age-dependency in the added worker effect.

2.5.1 Untargeted Moments

We begin this section by presenting untargeted life cycle profiles of individual labor market transitions in Figure 2.7. Again, in the model U comprises both the group of unemployed who receive benefits and those who exert costly search effort without receiving benefits.

First, consider transitions from employment over the life cycle (Figure 2.7a to 2.7c). The model captures that the likelihood of remaining in employment falls quite rapidly towards the end of working life, though the monthly transition probability out of employment never falls below 95%. The counterpart to this in model and data is a corresponding increase in the likelihood of moving from employment to out of the labor force. As agents get closer to the retirement age, it is not worthwhile for them to stay employed when they receive a bad match quality shock or have low human capital. By contrast, young agents continue to work even in these cases. Several model mechanisms account for this. First, young agents have a longer time horizon until retirement, so that they need labor income to cover consumption needs during working life. In contrast, old agents hold much higher levels of assets which they can use

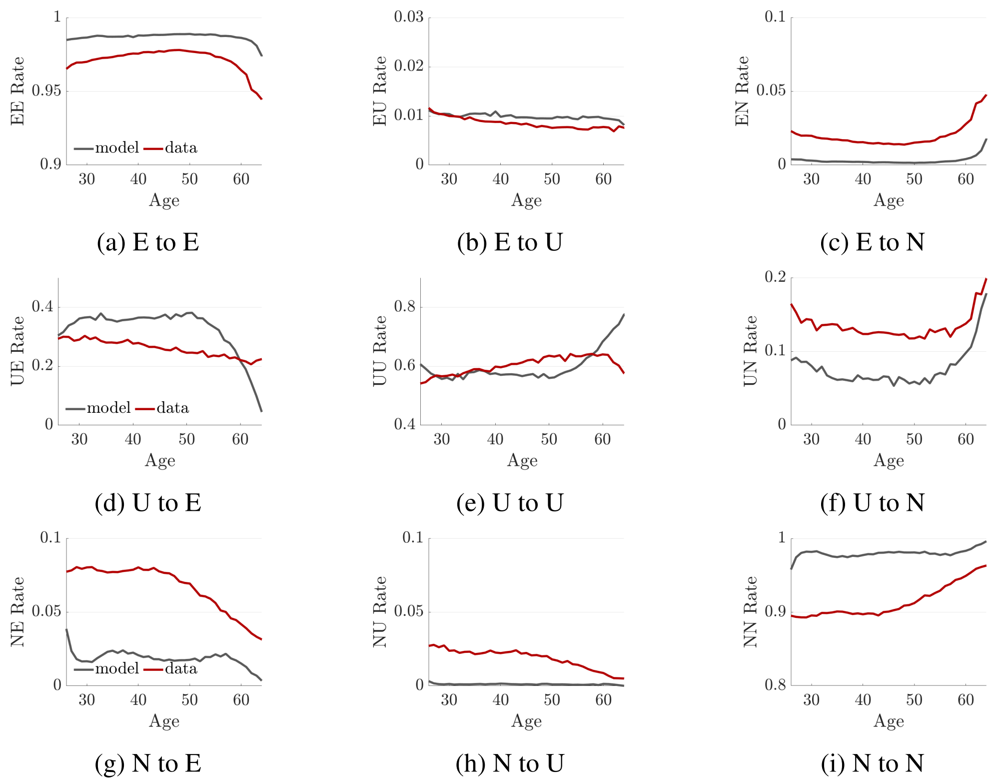


Figure 2.7 Labor Market Transitions over the Life Cycle

Notes: Figure 2.7 shows individual labor market transitions in the data and in the model. For the model, U includes both unemployed receiving benefits and searchers who do not receive benefits. The data is from the CPS.

to finance consumption. Second, human capital is only accumulated while employed. Thus, higher human capital is more valuable for the young as they can benefit from it for a longer time period. The model performs very well in matching the slightly decreasing path of E to U transitions over the life cycle.

Next, consider the transitions out of unemployment (Figure 2.7d to 2.7f). The model replicates that across the entire life cycle the most likely transition is to stay in unemployment. It also matches well that the probability of transitioning to employment decreases with age, whereas the probability of giving up on searching and leaving the labor force goes up with age. Finally, the model generates a fall in transitions from out of the labor force into employment (Figure 2.7g) but understates the likelihood to transition into unemployment (Figure 2.7h) over the life cycle, while it matches well the high persistence of non-participation (Figure 2.7i).

Again, it is apparent from these figures that the model generates too few transitions between out of the labor force and employment/unemployment. This is most likely due to the fact that we leave many important life events such as child birth, marital transitions, and health shocks unmodeled. We

Table 2.8 Joint Labor Market Transitions by Age (Model vs. Data)

	Primary earner transition	
	EE	EU/ES
Young (25-35):		
Cond. prob. of spousal NE transition	2.26%	3.12%
	6.66%	9.30%
Cond. prob. of spousal NS transition	0.40%	5.28%
	2.00%	6.89%
Cond. prob. of spousal NN transition	97.34%	91.60%
	91.34%	83.81%
Old (55-65):		
Cond. prob. of spousal NE transition	1.95%	2.24%
	4.29%	3.73%
Cond. prob. of spousal NS transition	0.11%	1.16%
	0.90%	2.75%
Cond. prob. of spousal NN transition	97.95%	96.60%
	94.81%	93.52%

Notes: This table compares joint labor market transitions by age in the model and in the data.

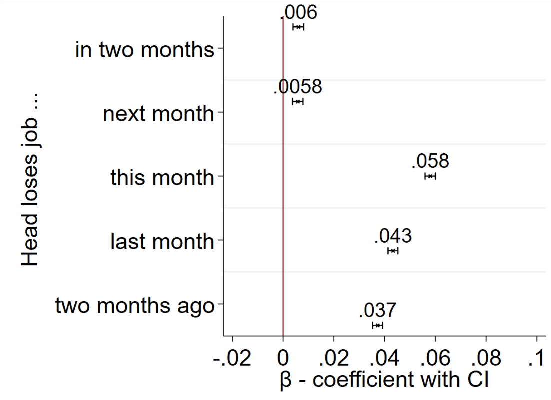
will show next, however, that the model captures well the impact of one key life event, job loss of the primary earner, on the labor force participation of out of the labor force spouses, the added worker effect.

2.5.2 The Added Worker Effect over the Life Cycle in the Model

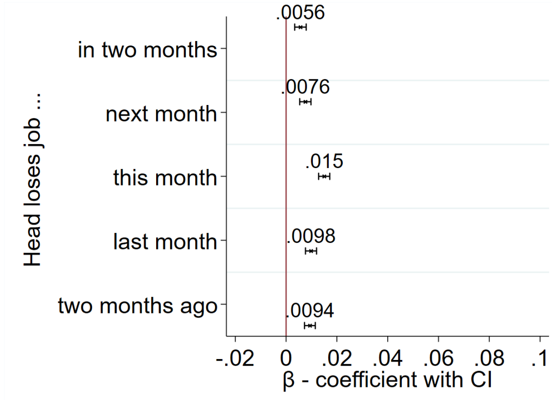
We now evaluate whether the model can replicate our main empirical finding: the age dependency in the added worker effect. To compare model to data, we replicate Table 2.3 from Section 2.2 with simulated model data in Table 2.8. For ease of comparison, we also report empirical transition probabilities.

For the young, the model is capable of producing a strong increase in the probability of moving from out of the labor force directly into employment and into unemployment upon job loss of the primary earner. The model generally underestimates the probability of spousal transitions directly into employment independently of the primary earner's transition. However, it captures very well the difference in probabilities depending on the primary earner transition, which is the added worker effect.

In the model, as in the data, there is a much smaller added worker effect for the old. The model reproduces that there is no substantially increased likelihood of transitioning from out of the labor force directly into employment when the primary earner loses a job for the old. Furthermore, the increased probability of searching for a job by exerting costly effort is much lower than for the young, in line with the data.



(a) Young: 25-35 years



(b) Old: 55-65 years

Figure 2.8 Dynamic Response: AWE by Age in the Model

Notes: Figure 2.8 shows the change in the probability that a non-participating spouse enters the labor force (either as unemployed or as employed) this month if household head loses/lost the job in two months, next month, this month, last month, two months ago, respectively, relative to the baseline in which the household head remains employed. Figure 2.8a shows the model results for young households; Figure 2.8b shows the model results for old households. The regression producing the coefficients is Equation (2.1).

Hence, the model performs well in generating the instantaneous added worker effect over the life cycle. To analyze anticipation effects and lagged responses, Figure 2.8 shows the results of replicating Equation (2.1) on model simulated output, separately by age. In line with the data, the model produces larger contemporaneous and lagged effects for the young than for the old. The lead effects are, however, of similar size across both age groups.

The model mechanisms that produce lagged responses are threefold. First, after becoming unemployed the primary earner may lose human capital which decreases potential human capital differences across spouses. Consequently, it may be optimal that both spouses search or to re-optimize on the actively searching household member. Second, unemployment benefits can expire, making employment a more desirable state. Third, households without any employed member may run down their assets to finance consumption, which increases the need to search for a new job to to re-accumulate assets for precautionary reasons and for retirement.

While the model produces some anticipation effect in the two months prior to a primary earner's job loss, these lead effects are smaller than in the data. Job loss is predictable because the exogenous separation probability depends on human capital. Spouses of low human capital employed individuals may enter the labor force because a future separation is relatively likely, whereas spouses of high human capital individuals choose not to do so because the chance of an exogenous separation is low. By the law of large numbers, these separations do in fact realize at higher rates for low human capital primary earners, producing the effect that spouses are more likely to enter the labor force in anticipation of a

Table 2.9 Joint Labor Market Transitions Counterfactuals

	Primary earner transition	
	EE	EU/ES
Young (25-35):		
Cond. prob. of spousal NE transition	2.26%	3.12%
Cond. prob. of spousal NS transition	0.40%	5.28%
Cond. prob. of spousal NN transition	97.34%	91.60%
Counterfactual meeting probabilities		
Cond. prob. of spousal NE transition	2.14%	2.93%
Cond. prob. of spousal NS transition	0.41%	5.36%
Cond. prob. of spousal NN transition	97.46%	91.71%
Counterfactual human capital		
Cond. prob. of spousal NE transition	1.70%	3.02%
Cond. prob. of spousal NS transition	0.24%	3.09%
Cond. prob. of spousal NN transition	98.06%	93.89%
Counterfactual assets		
Cond. prob. of spousal NE transition	0.11%	0.33%
Cond. prob. of spousal NS transition	0.11%	0.43%
Cond. prob. of spousal NN transition	99.78%	99.23%

Notes: This table shows the counterfactual joint labor market transition probabilities.

job loss. In addition, persistence in match quality might induce non-participating spouses to enter the labor force upon a decline in match quality for the employed spouse, preparing a potential future quit if match quality remains low.

2.5.3 Counterfactuals

Finally, we use the model to construct counterfactuals and analyze which channels are important in driving the age-dependency in the added worker effect. For that purpose, we start with the added worker effect of the young and then change individual model elements towards the counterparts of old households. Table 2.9 reports the results for three such counterfactuals together with the baseline results for young households.

The first counterfactual adjusts job arrival rates for young households. More specifically, we first compute the average job arrival rate for old and for young households in the model, restricting the sample to households with one member employed and one member out of the labor force. Afterwards, we adjust the individual arrival rates of each young household in our simulation by the difference between these previously computed means. This approach moves the average arrival rate of young households to that of their old counterparts, but preserves the relative distribution of arrival rates among the young. The second block of Table 2.9 shows that adjusting arrival rates has a limited impact on the added worker effect. This result arises because the average arrival rates for young and old are very similar: As most non-participating spouses are unlikely to accept a job offer, firms are only offering low arrival rates in order to satisfy their free entry condition. Nevertheless, the average arrival rate is slightly lower for older households resulting in fewer employment transitions both in the EE and in the EU case.[9]

In the second counterfactual, we adjust the human capital level of young households. Similar to above, we compute the difference in mean human capital levels across age groups separately for employed and non-participating spouses and adjust the human capital level of each young household by the difference. In our simulation, the employed spouse among older households has a higher human capital due to on average longer cumulative employment spells. In contrast, the human capital levels for non-participating spouses are very similar across age groups. This is partially driven by selection (low human capital individuals are more likely to be non-participating when they have an employed spouse) and partially by fast depreciation of human capital during non-employment in order to generate immediate wage losses from non-employment spells. Thus, the results of the second counterfactual can be attributed to a higher human capital level of the employed spouse during old age.

The third block of Table 2.9 shows that the increase in human capital of the employed spouse reduces transition probabilities into participation for both the EE and the EU case, but also dampens the added worker effect. When the human capital of a separated spouse is higher, this spouse is more likely to find a new job (arrival rates are increasing in human capital) and the difference in human capital levels across spouses is potentially larger, making a switch in the prime earner position less likely.

In a third counterfactual, we adjust the asset levels of young households in the same manner as arrival rates and human capital. Since old households have on average substantially higher asset levels we make all young households richer. The fourth block of Table 2.9 shows that this eliminates the incentive for a non-participating spouse to transition into participation. Hence, the added worker effect vanishes. Young households with asset holdings of the old are relatively rich for their age, reducing the incentive to work also in the baseline EE case, and are well insured against any labor market shock such that they do not have to rely on the added worker effect as a margin of insurance.

[9] This result may be partially due to the timing assumptions in the model. At the moment firms post vacancies in all the submarkets before separation shocks occur. Hence, out of the labor force spouses do not consider that their partner loses the job, translating into low acceptance probabilities and in turn low vacancy posting rates. In future work, we will investigate the robustness of the finding to different timing assumptions.

Taking all three counterfactuals together, we find that the substantially lower added worker effect among the old predominantly arises through higher wealth levels. Hence, older households exhibit a weaker AWE because they have better access to self-insurance through savings and are therefore less in need of other insurance margins, as opposed to a lack of opportunity to make use of the AWE.

2.6 Conclusion

In this paper, we provide evidence that the added worker effect is an important insurance margin against job loss of the primary earner for two-member households, but that the prevalence of this insurance channel strongly differs over the life cycle. When the primary earner loses a job, an out of the labor force spouse is much more likely to enter the labor force in order to offset the income loss compared to when the primary earner remains employed. In particular, this spousal labor supply response is very strong for young households and becomes continuously weaker as households age.

To analyze the mechanisms that drive this age-dependency, we build a stochastic life cycle model of two-member households with a frictional labor market. We calibrate the model economy to match salient features of the US labor market. The model endogenously generates the added worker effect and its decreasing magnitude over the life cycle. Model counterfactuals reveal that the added worker effect is weaker for old than for young households mainly because older households are better insured through larger asset holdings, so that their need for spousal insurance is lower. In addition, human capital of employed spouses is higher for the old, making the spousal labor supply less valuable, though this channel is quantitatively smaller. Differences in arrival rates across age groups contribute little to the difference in the added worker effect due to a general reluctance of firms to offer jobs to non-participating workers.

Chapter 3

Larger Transfers Financed with More Progressive Taxes? On the Optimal Design of Taxes and Transfers[1]

Abstract The U.S. fiscal system redistributes through a rich set of taxes and transfers, the latter accounting for a large part of the income of the poor. Motivated by this, we study the optimal joint design of transfers and income taxes. Within a simple heterogeneous-household framework, we derive analytical results on the optimal relationship between transfers and tax progressivity. Higher transfers are associated with lower optimal income tax progressivity. Redistribution is achieved with generous transfers while efficiency is preserved via a lower progressivity of income taxes. As such, the optimal tax-and-transfer system features larger progressivity of average than of marginal tax rates. We then quantify the optimal tax-and-transfer system in a rich incomplete-market model with realistic distributions of income, wealth, and income risk. The model features a novel flexible functional form for progressive income taxes and means-tested transfers. Relative to the current U.S. fiscal system, the optimal policy consists of more generous means-tested transfers, which phase-out at a slower rate. These larger transfers are financed with higher tax rates, but the taxes are not more progressive than the current system.

[1]This work was supported by computational resources provided by the BigTex High Performance Computing Group at the Federal Reserve Bank of Dallas.

3.1 Introduction

High economic inequality in the U.S. has put redistributive policies at the heart of the policy debate. The U.S. fiscal system redistributes through two main instruments: a progressive income tax and a rich set of means-tested transfers. Both components significantly alter the after-tax-and-transfer income distribution, as shown in Figure 3.1. Means-tested transfers account for about a fifth of households' income in the bottom income quintile. In contrast, income taxes paid by households in the top quintile reduce their income by more than a quarter.

In this paper, we study the optimal joint design of taxes and transfers. Both progressive income taxes and means-tested transfers are powerful tools to redistribute resources across households, but they also create efficiency costs. Large income tax progressivity implies high marginal tax rates at the top, which lower labor supply of the most productive households. Large transfers generate a heavy fiscal burden, to be financed with distortionary income taxes. Overall, what should be the joint optimal tax progressivity and generosity of transfers? Should transfers be more generous compared to the current U.S. system? If so, who should face higher tax rates to finance these larger transfers?

To answer these questions, we first build intuition in a simple, analytically tractable model, and then quantify the key trade-offs in a rich heterogeneous-agent model. A central insight throughout the analysis is that optimal income tax progressivity is decreasing in the size of transfers. This negative relationship between tax progressivity and transfers emerges for both redistribution and efficiency reasons. First, large transfers provide redistribution, reducing the value of a further reduction in inequality through more progressive taxes. Second, large transfers increase the fiscal burden; to increase tax revenues, it is efficient to incentivize high labor supply of the most productive households with lower marginal tax rates at the top. Quantitatively, given the large inequality in income and wealth in the U.S. we find that optimal transfers are large and phase-out slowly. The income tax schedule used to finance these large transfers is only moderately progressive, thereby preserving labor supply incentives and easing the financing of a large welfare state.

We start our analysis with an analytically tractable model, following Heathcote, Storesletten, and Violante (2017). A continuum of ex-ante homogeneous workers chooses consumption and labor supply subject to idiosyncratic income shocks, and a government has a loglinear tax schedule as its only policy instrument. In this case, a progressive tax schedule is desirable as the welfare gains of lower consumption inequality outweigh the efficiency costs of progressive taxation in the form of lower labor supply. We then extend this tractable framework by endowing the government with a lump-sum transfer as an additional instrument. We derive a closed-form formula for welfare based on local approximations

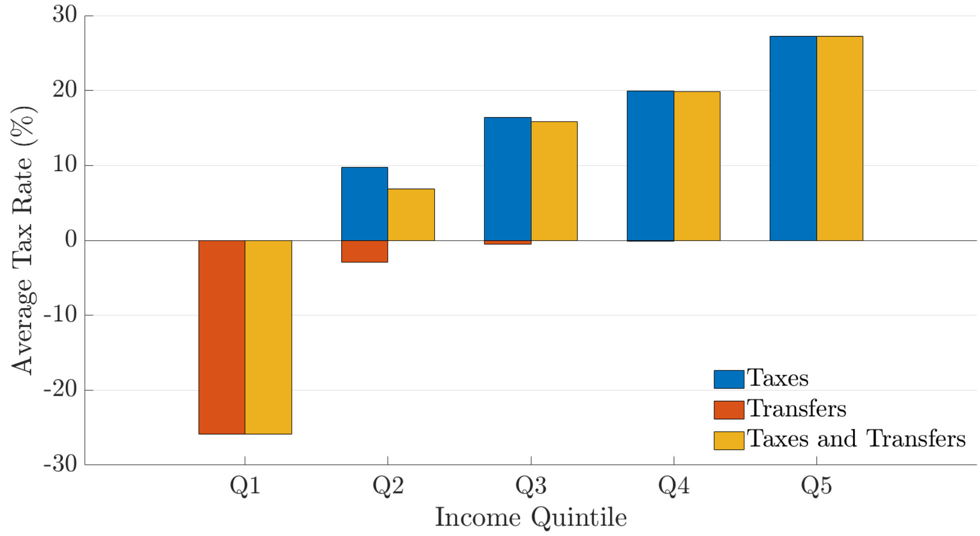


Figure 3.1 Redistribution in the United States

Notes: This figure shows average tax and transfer rates by income quintile for the working age population in the United States in 2013. Data is from the Congressional Budget Office (CBO). Details on the components of taxes and transfers are included in Appendix C.1.

around zero transfers. For realistic levels of income inequality, transfers should be positive. Higher transfers reduce the optimal income tax progressivity. This optimal negative relationship between income tax progressivity and transfers exists for both redistribution and efficiency reasons, as the welfare formula transparently shows. Lump-sum transfers already provide redistribution to the poor, reducing the redistributive gains of more progressive income taxes. Furthermore, larger transfers increase the fiscal burden, which raises efficiency concerns. These efficiency concerns are amplified by the fact that generous transfers are a force for lower labor supply themselves. Consequently, larger levels of spending on transfers are optimally financed with lower income tax progressivity to incentivize labor supply. While the analytical results are based on approximations, we quantitatively confirm them by numerically solving for the optimal combination of lump-sum transfers and loglinear taxes in the simple model. We find that redistribution should be achieved with generous transfers while efficiency should be preserved via less progressive income taxes. As such, the optimal tax-and-transfer system implies very progressive average tax rates while marginal tax rates actually turn out regressive.

Having established the intuition for the optimal relation between transfers and the progressivity of the tax rates schedule, we turn to our second question: What should the optimal level of transfers and the tax progressivity be relative to the current U.S. system? We answer this question using a quantitative heterogeneous agent model. For the quantitative analysis we propose new functional forms to closely approximate the two main components of the U.S. fiscal system: progressive income taxes and targeted transfers. In contrast to the loglinear specification, our proposed income tax function is bounded below by zero, in line with the statutory income tax rates in the United States. For the transfer function, we use a logistic functional form, which captures both the level and the phasing-out patterns of the current U.S. transfer system when appropriately parameterized. While tractable, these parameterized functional forms are flexible enough to explore a wide variety of shapes of the tax-and-transfer system. In particular, having a transfer, even of the lump-sum variety, allows to disentangle progressivity in marginal and average tax rates, which we showed to be important in the analytical model. Additionally allowing for phasing out of the transfers allows marginal tax rates to potentially be U-shaped, which is what is commonly optimal in the Mirrleesian optimal taxation literature.

For the quantitative analysis, we build an incomplete markets model à la Aiyagari (1994) with endogenous labor supply. We incorporate rich income dynamics with a Pareto tail for the productivity distribution to capture the concentration of income at the top and higher order moments of income risk. The tax reforms we consider in the quantitative model are once-and-for-all changes to the tax-and-transfer system. After a tax reform the economy slowly moves towards a new steady state and for the computation of welfare we take into account the entire transition. The optimal tax-and-transfer system in the quantitative model with realistic distributions of income, wealth, and income risk features large transfers that phase-out slowly, combined with moderately progressive taxes. Relative to the current U.S. system transfers are much larger. They are financed with higher taxes, which are not more progressive though than the current U.S. system. In line with the prescriptions from the theory, redistribution through very progressive average tax rates is achieved with less progressivity in marginal tax rates. In fact, because the phasing-out transfer causes high marginal tax rates at the bottom of the distribution marginal tax rates are falling over wide ranges of the income distribution. Welfare gains of the reform are large and the reform is supported by a vast majority of households.

Endowing the government with an income tax function and the transfer as an additional tool is essential to realize these welfare gains. Restricting the government to just the loglinear tax function yields substantial welfare losses relative to the optimal system with progressive taxes and a targeted transfer. With only the loglinear function achieving the substantial desired redistribution requires very high tax progressivity, which then also implies very high marginal tax rates at the top. Restricting the government to using a lump-sum transfer instead of a targeted transfer in combination with the income tax function is much less restrictive. Most of the welfare gains can also be achieved with a system of lump-sum transfers and relatively flat taxes. However, the amount of taxes that has to be raised is substantial, with marginal rates of more than 60% along the entire income distribution.

Related Literature. This paper builds on a large literature documenting rising inequality in the United States since the 1980s. Seminal contributions to this literature are Piketty and Saez (2003) on the rise of income inequality, Saez and Zucman (2016) on wealth inequality, and Piketty, Saez, and Zucman (2017) introducing the concept of distributional national accounts. Also on the empirical side, this paper is related to a number of contributions estimating parametric tax functions approximating the U.S. income tax system. Gouveia and Strauss (1994) propose a tax function and estimate it using U.S. data for the 1980s. Guner, Kaygusuz, and Ventura (2014) estimate several parametric tax functions using data for the year 2000. Feenberg, Ferriere, and Navarro (2020) provide estimates of the loglinear tax function over time. Splinter (2020) empirically analyzes the redistributive impacts of a variety of tax-and-transfer programs in the U.S.

On the theoretical side, we build on Heathcote, Storesletten, and Violante (2014) and Heathcote, Storesletten, and Violante (2017), who propose an analytical framework with partial insurance against idiosyncratic shocks to study risk sharing and optimal taxation. We extend this framework to allow for transfers. Our quantitative framework relates to several papers studying optimal tax progressivity in incomplete markets models. An early contribution to this literature is Conesa and Krueger (2006). More recently, several papers investigate optimal tax progressivity in frameworks with human capital accumulation (Badel, Huggett, and Luo, 2020; Krueger and Ludwig, 2016; Peterman, 2016), superstar earners (Kindermann and Krueger, 2021), transitional dynamics (Bakış, Kaymak, and Poschke, 2015), and with the goal of maximizing revenue (Guner, Lopez-Daneri, and Ventura, 2016). Our paper also relates to recent analyses of universal basic income policies and/or negative income taxes by Lopez-Daneri (2016), Daruich and Fernández (2020), Conesa, Li, and Li (2021), Luduvice (2021), and Guner, Kaygusuz, and Ventura (2021). While our model firmly falls in the Ramsey (1927) approach to optimal taxation, it is also related to some literature in the Mirrleesian tradition (Diamond, 1998; Mirrlees, 1971; Saez, 2001). Most papers in the Mirrleesian tradition use static models; some recent papers, however, extend the approach to dynamic models (Boerma and McGrattan, 2020; Farhi and Werning, 2013; Golosov, Troshkin, and Tsyvinski, 2016; Kapička, 2013). Our flexible functional forms allow for many shapes of marginal and average tax rates schedules, thereby helping to bridge the gap between the Mirrlees and the Ramsey approach. We share this aim with Findeisen and Sachs (2017).

Most closely related to us on the theoretical side is Heathcote and Tsujiyama (2021). They study the optimal Mirrleesian tax schedule in a static partial insurance economy and compare the optimal unrestricted tax schedule to simpler tax systems such as affine and loglinear tax schedules. Relative to them, while not solving for the optimal fully nonlinear plan, we endow the government with more flexible simple tools and incorporate them into a much richer quantitative model. Chang and Park (2020) compute the fully nonlinear optimal tax schedule in a Huggett economy, but restrict themselves to steady states. Our quantitative model is related to Boar and Midrigan (2021), who also study optimal taxation in a rich incomplete markets model. Their focus is, however, on the optimal progressivity of income and wealth taxes, whereas we focus on targeted transfer programs in combination with progressive income taxes.

Roadmap. The paper proceeds as follows. In Section 3.2, we introduce the simple model and characterize analytically the optimal interaction between lump-sum transfers and the progressivity of the income tax schedule. In Section 3.3, we build a rich quantitative model calibrated to the U.S. to quantitatively analyze the optimal joint design of the income tax schedule and means-tested transfers. Section 3.4 concludes.

3.2 An Analytical Model

As shown in Figure 3.1, transfers play an essential role in the redistribution of resources in the U.S. Motivated by that, we analyze the optimal progressivity of the tax schedule in the presence of transfers. We first present a simplified version of the partial-insurance framework of Heathcote, Storesletten, and Violante (2014, 2017), where a government has a loglinear tax schedule as its only policy instrument. We then extend this benchmark by endowing the government with a lump-sum transfer as an additional instrument. We derive analytical results based on local approximations around zero transfers. We confirm the analytical results by finding the global optimum using numerical methods.

3.2.1 Environment: A Static Bewley-Huggett Economy

The economy is populated by a continuum of ex-ante homogenous households, a representative firm, and a utilitarian government. Households are hand-to-mouth, value consumption c and leisure $1-n$, and their labor productivity z follows a Markov process. The representative firm uses a linear technology to transform labor into output. The government finances exogenous government spending G and a lump-sum transfer T with loglinear labor taxes.

Taxes. A household with labor income y pays (total) taxes $\mathscr{T}(y) = y - \lambda y^{1-\tau}$, where τ captures the progressivity and λ the level of taxes. For $\tau = 0$, tax rates are flat and equal to $1-\lambda$. When $\tau > 0$ ($\tau < 0$), marginal and average tax rates are increasing (decreasing) in income. Figure 3.2 shows the tax function. This tax function is widely used since at least Feldstein (1969). As it has been popularized in

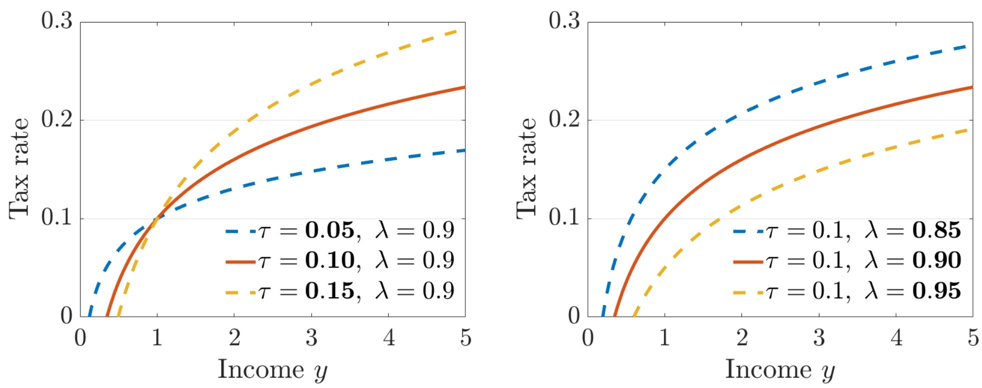


Figure 3.2 Loglinear Tax Function

Notes: This figure shows average tax rates given by the loglinear (HSV) tax function. In the left panel we vary the progressivity parameter τ; in the right panel we vary the level parameter λ.

the quantitative macroeconomics literature by Heathcote, Storesletten, and Violante (2017) we refer to it equivalently as loglinear or HSV tax function.

Households. In period t, household i chooses consumption c_{it} and labor n_{it} to maximize utility

$$u(c_{it}, n_{it}) = \log c_{it} - B \frac{n_{it}^{1+\varphi}}{1+\varphi} \tag{3.1}$$

subject to a static budget constraint

$$c_{it} = \lambda \left(z_{it} n_{it}\right)^{1-\tau} + T. \tag{3.2}$$

Consumption is equal to after-tax income, which is computed as the sum of income after applying the HSV tax function and a lump-sum transfer.

We assume a log-AR(1) process for z, with v_ω controlling the degree of heterogeneity across households

$$\log z_{it} = \rho_z \log z_{it-1} + \omega_{it}, \quad \omega_{it} \sim \mathcal{N}\left(-\frac{v_\omega}{2(1+\rho_z)}, v_\omega\right). \tag{3.3}$$

Two assumptions are key for tractability. First, households have a separable utility function and log utility over consumption such that income and substitution effects cancel out. Second, we assume that households are hand-to-mouth. Heathcote, Storesletten, and Violante (2017) assume a more complicated shock structure with a permanent uninsurable and iid insurable shocks. In their environment, a no trade theorem arises such that no savings are not an assumption but a result. In principle, we could use this shock structure as well. However, having transfers breaks the no trade theorem in any case and our analytical expressions for welfare below would become much more cumbersome, so we rely on this simpler assumption. We will relax the assumption of no savings in the quantitative model.

Government. The government finances exogenous government spending G and the lump-sum transfer T with the revenues from the loglinear tax function. The budget constraint reads

$$G+T=\int z_{it}n_{it}di-\lambda\int(z_{it}n_{it})^{1-\tau}di. \tag{3.4}$$

3.2.2 Welfare without Transfers

As a benchmark, we first consider the case in which the government does not have access to the lump-sum transfer. It can only use the loglinear tax function to raise revenues and redistribute. This is also the assumption in Heathcote, Storesletten, and Violante (2017), so that this is a special case of their results.[2]

Under the assumption that there is no transfer we can solve for welfare in closed form. We relegate the derivation to Appendix C.2.2. Here, we only outline the main steps. To obtain the expression for welfare, we derive the policy function for labor:

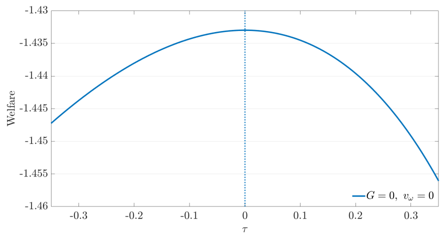
$$n_{it}=\frac{(1-\tau)}{B}^{\frac{1}{1+\varphi}}\equiv n_0(\tau). \tag{3.5}$$

Note that under these assumptions labor supply is only a function of preference parameters and tax progressivity τ. We can compute total labor supply and output from the individual policy functions. From the government budget constraint we can then compute the tax function level parameter λ. This implies consumption from the individual budget constraint. Knowing the policy functions for labor and consumption we can aggregate to total welfare as a function of tax progressivity:

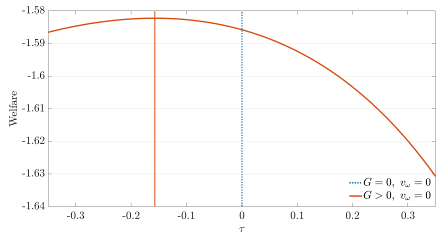
$$\mathscr{W}(\tau)=\log(n_0(\tau)-G)-\frac{1-\tau}{1+\varphi}-(1-\tau)^2\frac{v_\omega}{2(1-\rho_z^2)}. \tag{3.6}$$

These three terms have straightforward economic interpretations. The first two terms capture efficiency concerns. The first term is related to the size of the economy. With larger tax progressivity labor supply goes down, which reduces the size of the economy. This decreases welfare. At the same time, lower labor supply is beneficial, as households dislike working, which is captured by the second "labor

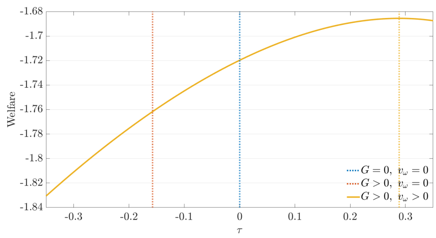
[2]Their environment is more general: In addition to the different shock structure, they consider skill investment and preference heterogeneity. We abstract from these features to focus attention on the economic significance of having transfers as an additional tool.



(a) No spending, no heterogeneity



(b) Spending, no heterogeneity



(c) Spending, uninsurable shocks

Figure 3.3 Welfare without Transfers

Notes: This figure shows welfare as a function of tax progressivity without a transfer.

disutility" term. The last term captures redistributive concerns. More progressivity compresses the distribution of consumption, which increases welfare because a utilitarian planner prefers a more equal distribution of consumption. This term gets more important the more dispersed incomes are.

We illustrate the forces determining optimal tax progressivity in Figure 3.3. For the numerical illustration, we set the inverse of the Frisch elasticity φ to 2.5 and the labor disutility parameter B such that labor supply $n_0(\tau) = 0.3$. We set $\tau = 0.18$, based on the estimates of Heathcote, Storesletten, and Violante (2017) for the U.S. We set government spending to match a spending to output ratio of 0.15. We fix $\rho_z = 0.935$ and set v_ω to match the variance of log consumption, also as measured in Heathcote, Storesletten, and Violante (2017).

In the first panel of Figure 3.3 we consider the special case in which government spending is zero and there is no heterogeneity. In this case, optimal tax progressivity is zero. There is no need to raise revenues and no need for redistribution, so it is best to not distort labor supply. In the second panel, we maintain the assumption of a representative agent but introduce the exogenous spending requirement. Then, optimal progressivity is negative. It is optimal to lower progressivity in order to increase labor supply and thereby make it easier to finance the spending requirement.

In these two cases with $v_\omega = 0$ setting progressivity optimally implements the first best allocations, which can be derived from maximizing utility subject to the resource constraint. First best labor supply

$n^\star(G)$ is such that $Bn^\varphi(n-G)=1$. Implementing this requires

$$\tau_0^\star(G) = -\frac{G}{n^\star(G)-G}, \tag{3.7}$$

which we show in Appendix C.2.1. As shown in Figure 3.3, optimal progressivity is zero when exogenous spending is zero and negative when there is a positive spending requirement.

Panel 3 of Figure 3.3 shows the optimal progressivity when we add heterogeneity. Now the planner has a motive for increasing progressivity to redistribute. Then, optimal progressivity turns positive such that average and marginal tax rates are increasing in income.

3.2.3 Welfare with Transfers

Having established the benchmark in which the government can only use the loglinear income tax function, we now turn to the case in which it additionally has the lump-sum transfer at its disposal. In that case, we can no longer express the labor policy function in closed form. Instead we approximate the labor policy function by applying the implicit function theorem to the first order condition around the case of a zero transfer. The approximated policy function reads

$$\hat{n}_{it} \approx n_0(\tau) - \frac{T}{1+\varphi}\frac{n_0(\tau)}{n_0(\tau)-G}\eta^{\frac{-\tau}{2}} z_{it}^{-(1-\tau)}, \tag{3.8}$$

where $\eta \equiv \exp\left((1-\tau)\frac{v_\omega}{1-\rho_z^2}\right)$. Labor supply is falling in the size of the transfer due to the wealth effect. With this approximation we can follow similar steps as in the case without transfers to obtain an expression for welfare as a function of progressivity τ and transfer T:

$$W(\tau,T) = W(\tau,0) + \frac{T}{1+\varphi}\frac{\eta^{-\tau}}{n_0(\tau)-G}\left(-\frac{n_0(\tau)}{n_0(\tau)-G} + (1-\tau)\eta + (\varphi+\tau)(\eta-\eta^\tau)\right). \tag{3.9}$$

The welfare expression contains the expression for welfare from the case without transfers but is extended by terms that depend on the presence of the transfer.

Consider first again the representative agent case, in which $v_\omega = 0$ and $\eta = 1$. For any T, there is an optimal τ that implements the first best. This relationship, also derived in Appendix C.2.1, is described by

$$\tau(G,T) = \frac{-(G+T)}{n^\star(G)-(G+T)}. \tag{3.10}$$

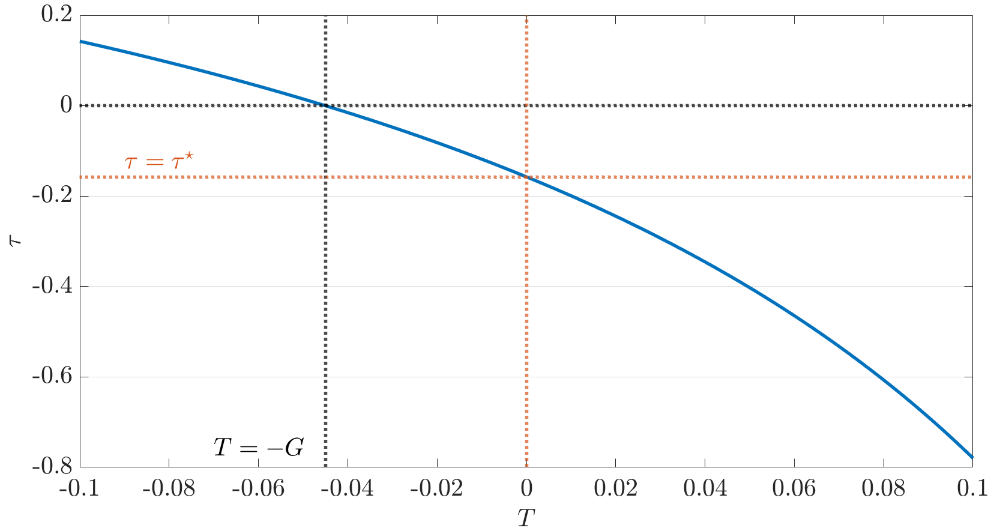


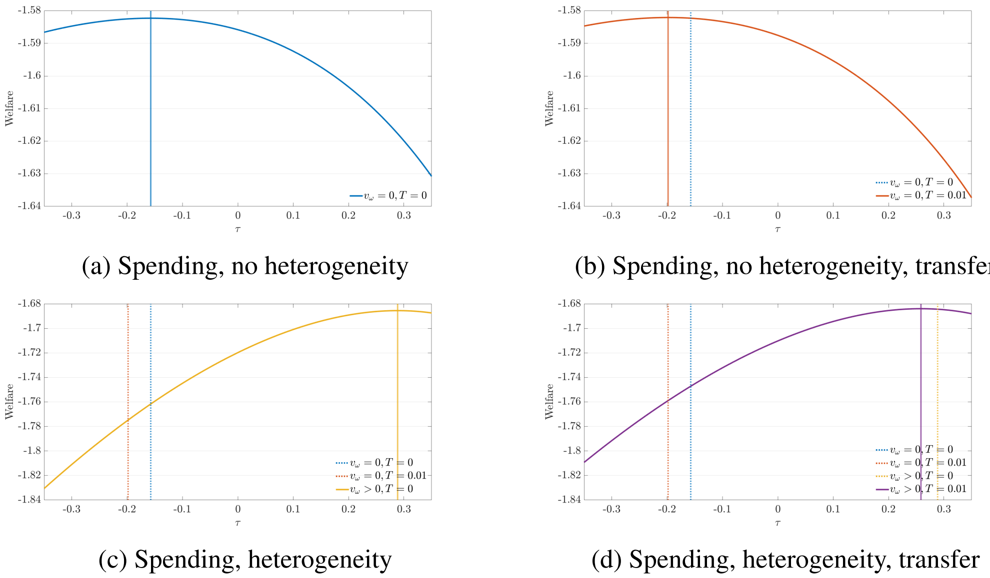
Figure 3.4 Optimal Progressivity with Transfers (Representative Agent): First Best

Notes: This figure shows optimal progressivity of the loglinear income tax as a function of the size of the lump-sum transfer.

and is plotted in Figure 3.4. First note that in the figure we highlight two special cases. The first special case is the one where transfers are zero, as discussed in the previous section. At zero transfers optimal progressivity is negative for efficiency reasons because negative progressivity incentivizes labor supply easing the financing of the exogenous spending.The second special case we highlight in the figure is at $T = -G$. The negative transfer, i.e. a lump-sum tax, increases labor supply due to the wealth effect. Hence, it is not necessary anymore to have negative progressivity to incentivize higher labor supply: At this specific level of transfer, the optimal progressivity is zero even though there is a positive spending requirement.

More generally, the figure shows a negative optimal relationship between the transfer and progressivity τ to implement the first best allocation. At $\tau = \tau_0^\star(G)$ the transfer is optimally zero. For all larger values of τ, the transfer is negative; for all smaller values, the transfer is positive. The efficiency gains from T are decreasing in τ.

Considering the general case with heterogeneity, this efficiency motive for a negative relationship between T and τ remains at work. It is captured by the first two terms within the bracket. Additionally, now there are redistributive concerns. These are captured by the final term in the bracket. Generally, the lump-sum transfer is good for redistribution (the term is positive) because it reduces consumption inequality. However, the redistribution gains from having the transfer are decreasing in τ. This is most easily seen by considering the extreme case of $\tau = 1$, in which after tax incomes are equalized. Then, the



(a) Spending, no heterogeneity

(b) Spending, no heterogeneity, transfer

(c) Spending, heterogeneity

(d) Spending, heterogeneity, transfer

Figure 3.5 Welfare with Transfers

Notes: This figure shows welfare as a function of tax progressivity with a transfer.

redistributive gains from having a positive transfer are zero. Hence, redistribution concerns strengthen the negative optimal relationship between transfers and progressivity of the income tax function that is already given for efficiency reasons.

We illustrate how the introduction of a small transfer affects optimal progressivity of the tax function in Figure 3.5. We start in the first panel from the scenario with spending but no heterogeneity. As discussed above, optimal progressivity is negative there. When we introduce a small transfer in the second panel, the optimal progressivity falls. Because this is the representative agent case with no redistributive motive at play, this is solely for efficiency reasons. In the third panel we show again the case with spending and heterogeneity in which positive progressivity is optimal. In the fourth panel we again add a small transfer leading to a lower optimal τ, for efficiency and redistribution reasons.

3.2.4 Optimal Taxes and Transfers: Global Solution

So far we have used the expression for welfare to establish the optimal relationship between T and τ. Since it is based on an approximation around the case of a zero transfer we cannot use the formula to compute the optimal combination of transfer and loglinear tax function. Hence, we now compute the global optimum numerically, which we can also use to judge the quality of the approximation. Figure 3.6 shows the optimal τ as a function of T/Y. Welfare approximated with the formula provides a very good fit for the optimal relationship between transfers and tax progressivity. At zero transfer the formula and the global solution give the exact same solution by construction. However, the formula

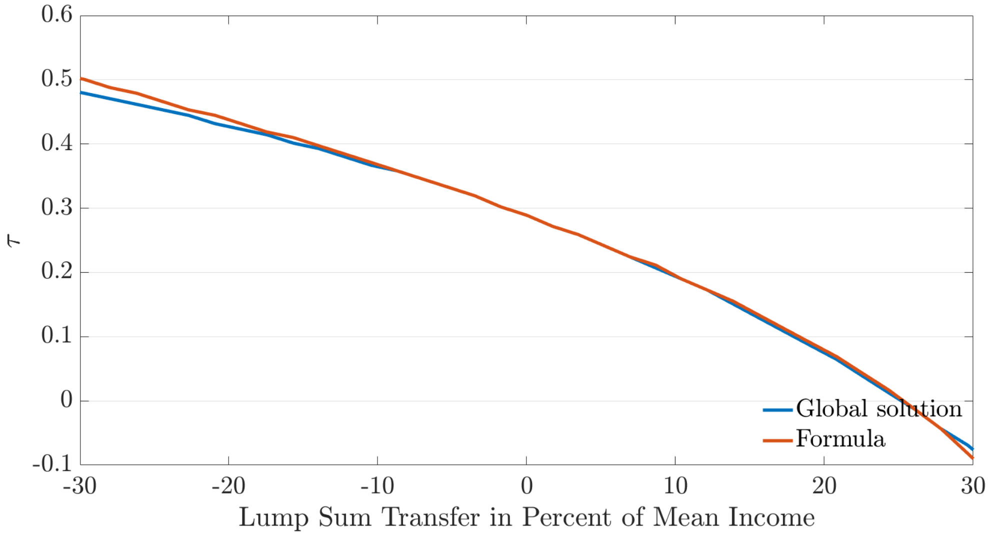


Figure 3.6 Optimal Progressivity: Quality of Approximation

Notes: This figure compares optimal progressivity as a function of the transfer given by the approximated formula to the global solution.

provides a very good estimate for the optimal progressivity given T for a wide range of transfers. Only for very large transfers does the quality of the approximation deteriorate.

Figure 3.7 shows again the optimal relationship between the transfer and progressivity from the global solution and the optimal combination of the two. As we have established before, the optimal τ is falling in T. The highest welfare is achieved with a very sizeable lump-sum transfer, in combination with negative progressivity. The transfer amounts to roughly 30% of mean income. At the same time, the progressivity of the income tax function is -0.07.

In Figure 3.8 we plot average and marginal tax rates implied by the tax-and-transfer system. The large lump-sum transfer implies very negative average tax rates at the bottom of the income distribution, which are then increasing towards zero as incomes rise. The average tax rate implied by the tax function alone is highest at the bottom of the income distribution and lower at the top. Overall, however, the tax-and-transfer system is very progressive in average tax rates. This is very different for marginal tax rates. Since the transfer is lump-sum it implies a marginal tax rate of zero everywhere. Because the income tax code is regressive, marginal tax rates are falling over the entire income distribution. Thus, the optimal plan combines very progressive average tax rates, implying significant redistribution, with regressive marginal tax rates, preserving labor supply incentives.

For comparison, the figure also includes the average and marginal rates of the optimal loglinear plan when we force the transfer to be zero. Progressivity is positive and high ($\tau = 0.29$) in this case

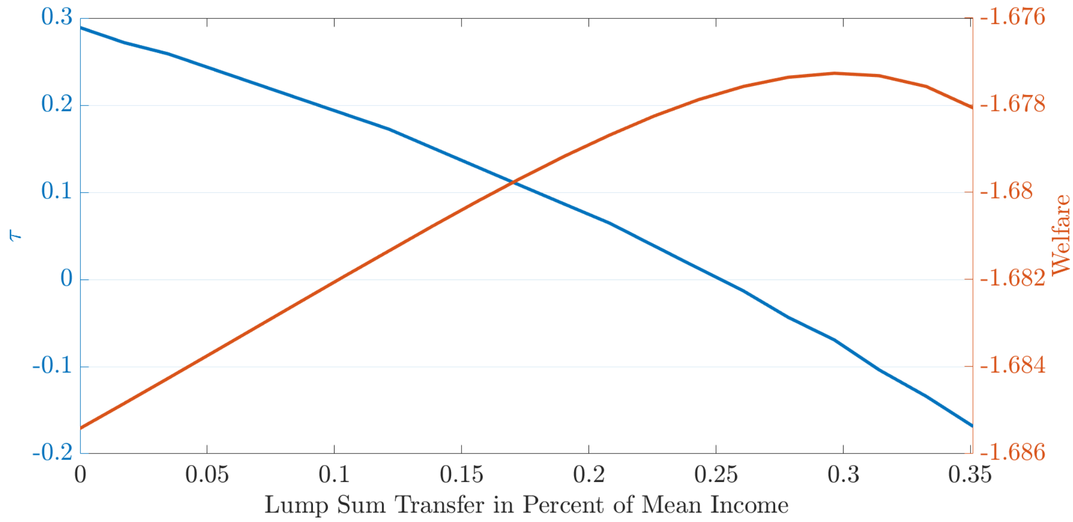


Figure 3.7 Optimal Tax-and-Transfer System

Notes: This figure shows optimal tax progressivity as a function of the lump-sum transfer. It also shows the welfare achieved at a given lump-sum and the associated optimal income tax progressivity.

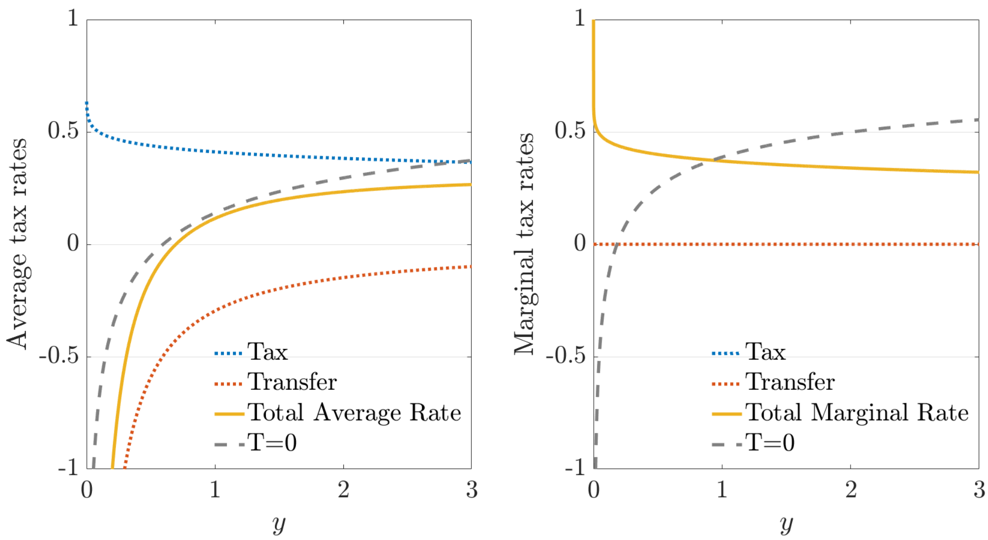


Figure 3.8 Optimal Tax-and-Transfer System: Average and Marginal Rates

Notes: This figure shows average and marginal tax rates given by the optimal tax-and-transfer system. It also compares the average and marginal rates of the optimal loglinear tax function when we force transfers to be zero.

because the planner wants to redistribute resources. However, the functional form requires that to achieve redistribution through increasing average tax rates marginal tax rates also have to be increasing. Marginal tax rates at the top of the income distribution are much higher in this scenario. Comparing the average and marginal tax rates schedules illustrates a key insight from this analysis: It is generally preferable to have lower progressivity in marginal tax rates than in average tax rates. While the optimal level of the transfer and the optimal associated tax progressivity depends on fundamentals such as the size of exogenous spending or the shape of the income distribution, it will always be true also in the quantitative model that having more progressive average than marginal tax rates is optimal.

3.2.5 Taking Stock

In summary, there are two main takeaways from the analytical part. First, there is optimally a negative relationship between the size of government transfers and the progressivity of the taxes which are used to finance the transfers. This negative relationship is due to both efficiency and redistribution reasons. Second, adding a transfer to the loglinear tax function is welfare improving because it allows to achieve more progressivity in average than in marginal rates of the entire tax-and-transfer system. The analytical model suggests that it is optimal to combine very progressive average tax rates with regressive marginal rates. It is, however, too simple to make a truly quantitative statement about the optimal combination of the instruments. To address that question, we move to a more quantitative macro model of the U.S. economy next.

3.3 Quantitative Model

We now turn to the quantitative model in order to investigate how generous transfers and how progressive the associated taxes should be in an economy calibrated to the U.S., with realistic distributions of income, wealth, and income risk. For that purpose, we build an Aiyagari (1994) variant of the incomplete markets model: In contrast to the simple model, production uses capital in addition to labor, the government uses debt, and households can save to self-insure against idiosyncratic shocks.

3.3.1 Environment

The quantitative model is an incomplete-market model in the tradition of Bewley (1977), Huggett (1993), and Aiyagari (1994) with endogenous labor supply as in Pijoan-Mas (2006). Time is discrete and indexed by $t = 0, 1, 2, \ldots$. The economy is populated by a continuum of households, a representative firm, and a government. The firm has access to a constant returns to scale technology in labor and capital given by $Y = K^{1-\alpha}L^{\alpha}$, where K, L, and Y stand for capital, labor, and output, respectively. Both factor inputs are supplied by households. We assume constant total factor productivity.

Households. Households have preferences over sequences of consumption and hours worked given as follows:

$$\mathscr{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\sigma}}{1-\sigma} - B \frac{n_t^{1+\varphi}}{1+\varphi} \right], \tag{3.11}$$

where c_t and n_t stand for consumption and hours worked in period t. Households have access to a one period risk-free bond, subject to a borrowing limit $\underline{a}$. Their idiosyncratic labor productivity z follows a Markov process with transition probabilities $\pi_z(z', z)$.

Let $V(a,z)$ be the value function of a worker with level of assets a and idiosyncratic productivity z. Then,

$$\begin{aligned} V(a,z) = & \max_{c,a',n} \left\{ \frac{c^{1-\sigma}}{1-\sigma} - B \frac{n^{1+\varphi}}{1+\varphi} + \beta \mathbb{E}_{z'} \left[V\left(a', z'\right) | z \right] \right\} \\ & \text{s.t.} \\ & c + a' \leq wzn + (1+r)a - \mathscr{T}(wzn, ra) \\ & a' \geq 0, \end{aligned} \tag{3.12}$$

where w stands for wages and r for the interest rate. We impose an exogenous borrowing limit of zero. Households face a distortionary tax $\mathscr{T}(wzn, ra)$, which depends on labor income wzn and capital earnings ra separately. We will be more specific on the concrete functional forms for taxes and transfers below. Every period, households face the problem in (3.12) and make optimal labor, consumption, and saving decisions accordingly. Let $n(a,z), c(a,z)$, and $a'(a,z)$ denote the optimal policies.

Firms. Every period, the firm chooses labor and capital demand in order to maximize current profits,

$$\Pi = \max_{K,L} \left\{ K^{1-\alpha}L^{\alpha} - wL - (r+\delta)K \right\}, \tag{3.13}$$

where δ is the depreciation rate of capital. Optimality conditions for the firm are standard: Marginal productivities are equalized to the cost of each factor.

Government. The government's budget constraint is given by:

$$G + (1+r)D = D + \int \mathscr{T}(wzn, ra) d\mu(a,z), \tag{3.14}$$

where D is government's debt and $\mu(a,z)$ is the measure of households with state (a,z) in the economy. Government spending G is kept constant.

Stationary Equilibrium Definition. Let A be the space for assets and X the space for productivities. Define the state space $S = A \times X$ and $\mathscr{B}$ the Borel $\sigma-$algebra induced by S. A formal definition of the competitive equilibrium for this economy is provided below.

A **recursive competitive equilibrium** for this economy is given by: value function $V(a,z)$ and policies $\{n(a,z), c(a,z), a'(a,z)\}$ for the household; policies for the firm $\{L,K\}$; government decisions $\{G,B,\mathscr{T}\}$; a measure μ over $\mathscr{B}$; and prices $\{r,w\}$ such that, given prices and government decisions: *(i)* Households' policies solve the household problem and achieve value $V(a,z)$, *(ii)* Firm's policies solve its static problem, *(iii)* Government's budget constraint is satisfied, *(iv)* Capital market clears: $K + D = \int_{\mathscr{B}} a'(a,z) d\mu(a,z)$, *(v)* Labor market clears: $L = \int_{\mathscr{B}} zn(a,z) d\mu(a,z)$, *(vi)* Goods market clears: $Y = \int_{\mathscr{B}} c(a,z) d\mu(a,z) + \delta K + G$, *(vii)* The measure μ is consistent with households' policies: $\mu(\mathscr{B}) = \int_{\mathscr{B}} Q((a,z),\mathscr{B}) d\mu(a,z)$ where Q is a transition function between any two periods defined by: $Q((a,z),\mathscr{B}) = \mathbb{I}_{\{a'(a,z)\in\mathscr{B}\}} \sum_{z'\in\mathscr{B}} \pi_z(z',z)$.

3.3.2 Calibration

We calibrate the model to the U.S. economy in 2013. We focus on matching well the tax-and-transfer system and the distributions of income, wealth, and income risk, in order to appropriately capture the redistribution and insurance needs in the U.S. Before we discuss the data moments and parameter choices we introduce our new functional forms for taxes and transfers and discuss the income process.

Tax-and-Transfer System. We endow the government with two fiscal tools capturing the key elements of the U.S. tax-and-transfer system. The first function is a new tax function, which is similar to the loglinear tax function, but importantly implies positive rates for all income levels. We prefer this tax function for our quantitative analysis, as we model transfers separately, while the loglinear tax function is supposed to capture the entire tax-and-transfer system and therefore turns negative at low income levels. The second tool the government can use is a targeted transfer function. In contrast to the simple model in the previous part, we do not just allow for a lump-sum transfer, but for a transfer that phases out with income, in line with transfer programs in the U.S.

The new income tax function is given by

$$\mathscr{T}(y_\ell, y_k) = \exp\left(\log(\lambda)\left(\frac{y_\ell}{\bar{y}}\right)^{-2\theta}\right) y_\ell + \tau_k y_k. \tag{3.15}$$

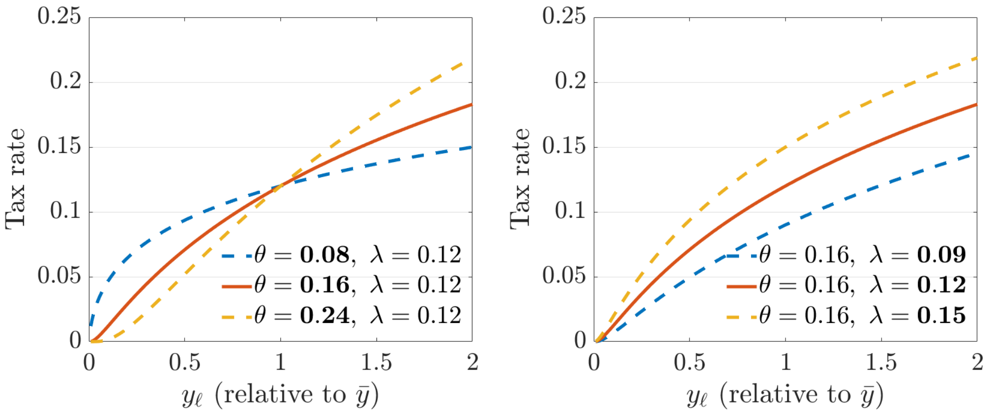


Figure 3.9 New Income Tax Function

Notes: This figure shows average tax rates given by the new income tax function. In the left panel we vary the progressivity parameter θ; in the right panel we vary the level parameter λ.

Capital income is taxed linearly at rate τ_k. We keep this linear capital income tax rate for all our optimizations. Labor income is taxed with the new tax function, which is illustrated in Figure 3.9. The parameter λ captures the overall level of tax rates. At income level $y_\ell = \bar{y}$ the average tax rate is λ. The parameter θ captures the progressivity of the tax code. The higher θ the more progressive is the income tax system. A θ of zero corresponds to a flat tax, and a negative θ implies regressive tax rates. In Figure 3.10 we compare the new income tax function to the loglinear tax function. The figure illustrates the key difference: The HSV tax function turns negative at low income levels, whereas our new tax function always implies positive taxes, in line with statutory tax rates in the U.S.

In addition to income taxes, we model targeted transfers. The functional form for transfers is

$$T(y) = m\bar{y}\frac{2\exp\left\{-\xi\left(\frac{y}{\bar{y}}\right)\right\}}{1+\exp\left\{-\xi\left(\frac{y}{\bar{y}}\right)\right\}}. \tag{3.16}$$

The parameter m governs the generosity of the transfer. Specifically, m is the level of the transfer at zero income, expressed relative to median income. The effect of changing m is shown in the left panel of Figure 3.11. The parameter ξ determines how quickly the transfer is phased-out with income. A higher ξ implies a quicker phase-out. This is shown in the right panel of Figure 3.11. This functional form is motivated by the fact that income security programs in the U.S. are means tested. Furthermore, while the lump-sum allows for breaking the tight link between average and marginal tax rates implied by just the tax rates schedule, the phasing out of the transfer allows for non-monotonic marginal rates of the entire tax-and-transfer system, as will become clear when we look at the optimal fiscal system below. Note that while the income tax function applies to labor and capital income separately, the

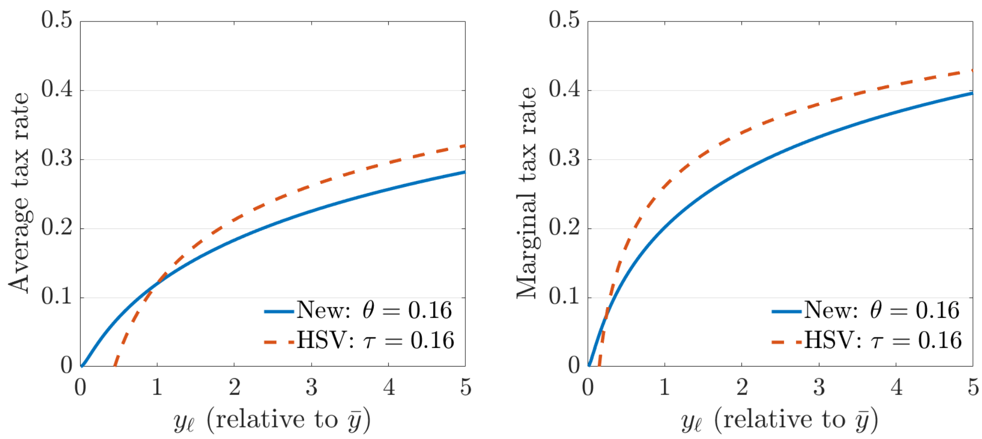


Figure 3.10 New Income Tax Function: Comparison to Loglinear Function

Notes: This figure compares average and marginal tax rates implied by the loglinear tax function and the new tax function.

transfer function conditions on total income. Empirically, this is motivated by the fact that most transfer programs are also asset tested.

Income Process. We assume that household productivity follows a Gaussian Mixture Autoregressive (GMAR) process:

$$\log z_t = \rho \log z_{t-1} + \eta_t$$
$$\eta_t \sim \begin{cases} \mathcal{N}\left(\mu_1, \sigma_1^2\right) & \text{with probability } p_1, \\ \mathcal{N}\left(\mu_2, \sigma_2^2\right) & \text{with probability } 1 - p_1. \end{cases} \tag{3.17}$$

Traditionally, most macroeconomic heterogeneous agent models have assumed that shocks are drawn from a normal distribution. This assumption implies that many households experience medium sized productivity shocks very often and that very small changes and very large changes are relatively rare. This is at odds with empirical evidence leveraging population wide data on earnings growth from Social Security records, as shown in Guvenen, Karahan, Ozkan, and Song (2021). They show that earnings growth exhibits stark deviations from normality: The earnings growth distribution is negatively skewed and exhibits excess kurtosis. Relative to a normal distribution there are more individuals with very small and very large earnings changes, but much fewer with medium sized earnings changes. Often times, the very large earnings changes tend to be negative. These features of the earnings growth distribution can be captured in a simple way using the GMAR process. Households draw shocks from a mixture of two normal distributions, one of which has a much higher variance than the other. It is more likely to

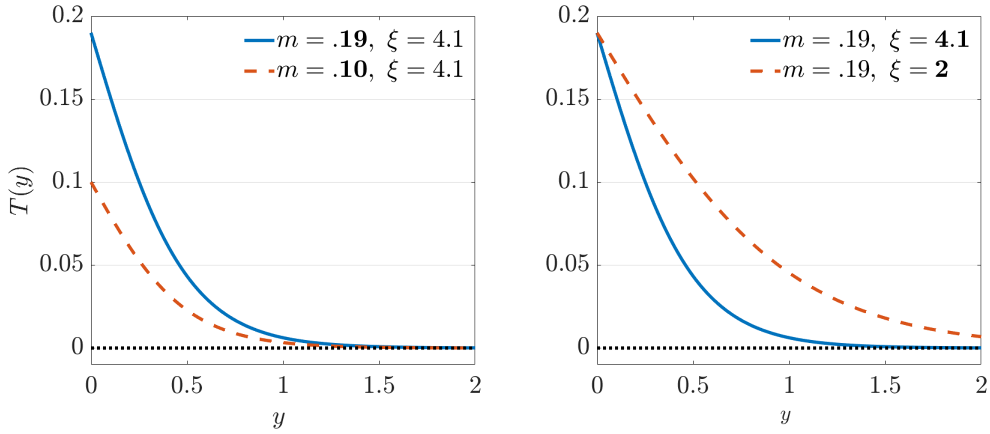


Figure 3.11 New Targeted Transfer Function

Notes: This figure illustrates the shape of the transfer function for different parameter combinations. In the left panel we vary the level parameter m; in the right panel we vary the phasing-out parameter ξ.

draw a shock from the low variance distribution such that households experience small shocks most of the time. However, if they draw from the normal with the high variance there is a high probability of a large shock. By having a positive mean for the low variance normal and a negative mean for the high variance distribution we can capture the negative skewness of the earnings growth distribution. We discretize the productivity process using the method of Farmer and Toda (2017).

We make one additional adjustment to the productivity process to better capture the concentration of incomes at the top. As Hubmer, Krusell, and Smith (2020) we adjust the top income states such that they follow a Pareto instead of a lognormal distribution. Specifically, we adjust the productivity states of the top 15%. To do so, we assume that they follow a Pareto distribution with tail parameter κ.

Targets and Parameters. We now describe how we choose the parameters of the model. In a first step, we fix a number of parameters exogenously. First, we set the coefficient of relative risk aversion σ to two. While we used log utility in the theory for tractability, a larger coefficient of risk aversion is more standard in the literature. We also fix the inverse Frisch elasticity φ to 2.5. We set the production side parameters to standard values, with the labor share $\alpha = 0.64$ and the depreciation rate $\delta = 0.08$. We fix the capital tax rate $\tau_k = 0.35$ (Trabandt and Uhlig, 2011) and keep it constant at this value in all our tax optimizations. Finally, we set the Pareto tail parameter κ to 1.6 (Aoki and Nirei, 2017). These and all other parameter values are summarized in Table 3.1.

The remainder of the parameters is calibrated internally. For the income process we have to choose five parameters $(\rho, \mu_1, \sigma_1, \sigma_2, p_1)$. Given these choices μ_2 is implied from the restriction that the mean of the process is zero. We target five moments from the earnings growth distribution and the earnings

Table 3.1 Parameter Values

Parameter	Interpretation	Value
Preferences		
β	Discount factor	0.962
σ	Risk aversion	2.000
φ	Labor supply elasticity	2.500
B	Disutility of labor	85.000
Income Process		
ρ	Persistence	0.935
p_1	Weight on first normal	0.850
p_2	Weight on second normal	0.150
μ_1	Mean of first normal	0.016
μ_2	Mean of second normal	-0.091
σ_1	Std. dev. of first normal	0.150
σ_2	Std. dev. of second normal	0.610
κ	Pareto tail parameter	1.600
Production		
α	Labor share	0.640
δ	Depreciation rate	0.080
Government		
D	Public debt	0.600
G	Government spending	0.126
θ	Tax progressivity	0.160
λ	Tax level	0.118
τ_k	Capital tax rate	0.350
m	Transfer level	0.190
ξ	Transfer phase-out	4.100

Notes: Table 3.1 summarizes the parameter values for the calibrated steady state economy.

distribution to discipline these parameters. An agent in our model is a household. Therefore, we cannot directly target the data moments on the earnings growth distribution reported in Guvenen, Karahan, Ozkan, and Song (2021), which are computed for individuals rather than households. Instead we compute household labor earnings growth from the Panel Study of Income Dynamics (PSID) using data from 1978 to 1992.[3] As is standard in the literature, we impose a minimum earnings threshold of $1,500 to only consider households with some labor market attachment. The moments we target are the standard deviation (0.35), the difference between the 90th and the 10th percentile (0.64), the skewness (-0.45), and the kurtosis (12) of the household earnings growth distribution. The final moment we target with the productivity process is the income share of the top 10% of the income distribution.

[3]We use these early years of the PSID because the PSID became biannual in the late 1990s and labor income definitions changed after 1992.

Table 3.2 Calibration: Income and Wealth Distributions

Data	Q1	Q2	Q3	Q4	Q5	Top 10
Labor income	2%	9%	15%	23%	52%	38%
Net worth	-1%	1%	3%	9%	88%	71%
Baseline	Q1	Q2	Q3	Q4	Q5	Top 10
Labor income	4%	9%	14%	20%	52%	38%
Net worth	0%	2%	8%	18%	72%	52%

Notes: Labor income shares by labor income quintiles and wealth shares by wealth quintiles, households aged 25-60. Data: PSID 2013 for labor income; SCF 2013 for wealth and top-10 labor income.

We compute this from the Survey of Consumer Finances (SCF) instead of the PSID because the SCF oversamples the rich, thereby providing a better view of the incomes at the top of the distribution.

The fit for the income and wealth distributions is shown in Table 3.2. We match the top 10% income share by construction, but the fit for all income quintiles is very good. The model performs a bit less well for wealth inequality. While it is able to generate a significant amount of wealth concentration, with more than half of all wealth in the economy held by the top 10% of the wealth distribution, it does not account for the extreme concentration of wealth in the data. In the model the top quintile wealth share is 72% compared to 88% in the data. Quintiles 3 and 4 specifically hold too much wealth relative to the data in the model. To improve the model fit in this dimension we later consider a model extension with heterogeneous stochastic discount factors.

To parameterize the tax-and-transfer system we have to choose four parameters, level and progressivity of the tax function and level and phase-out of the transfer function. We choose to match the tax-and-transfer rates from the CBO that we have shown in the introduction in Figure 3.1. The parameters of the tax-and-transfer function are summarized in Table 3.1, as are all other parameters. In Table 3.3 we show average tax rates and average transfer rates by income quintile in the model and compare it to the CBO data. Given that we have only two parameters for the tax function we cannot match the tax rates for all income quintiles perfectly. We choose to match particularly closely the average tax rates in the second and fifth income quintile, which implies that tax rates are too high at the bottom and slightly too low in quintiles three and four. Transfer rates are matched well, accounting for a significant share of income in the bottom quintile but then phasing-out quickly.

We also depict average and marginal tax rates implied by the tax-and-transfer system in Figure 3.12, as a function of labor income. For this figure, we assume a capital income of zero. This does not matter for the tax function, but does matter for the transfer function. For higher capital incomes, the marginal tax rates implied by the transfer function are lower, as the marginal tax rates are falling in

Table 3.3 Calibration: Average Tax and Transfer Rates

Data	Q1	Q2	Q3	Q4	Q5
Tax rate	0%	10%	16%	20%	27%
Transfer rate	26%	3%	1%	0%	0%
Model	Q1	Q2	Q3	Q4	Q5
Tax rate	8%	11%	14%	17%	28%
Transfer rate	24%	4%	1%	0%	0%

Notes: Average tax rates paid and transfer rates received per income quintile. Data: CBO 2013, working-age households.

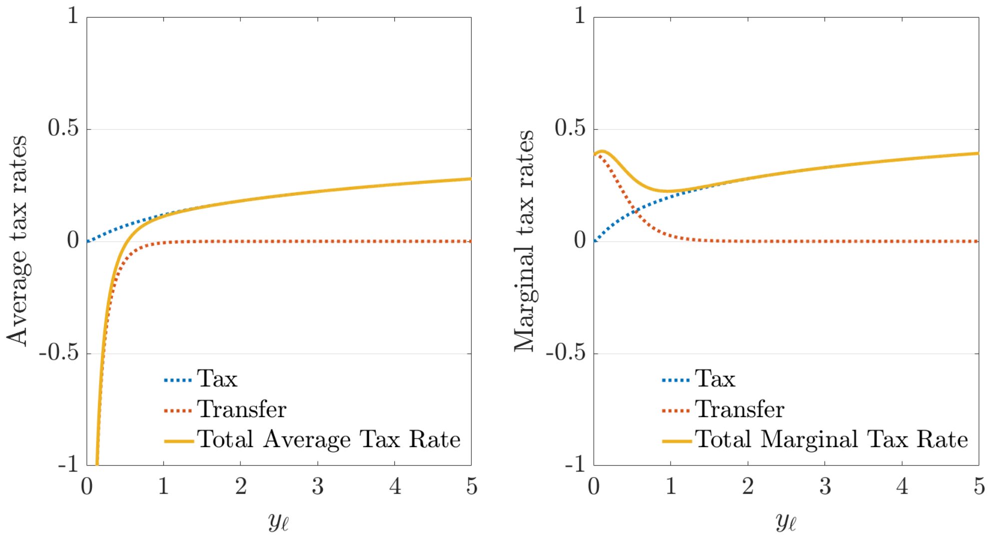


Figure 3.12 Calibration: Average and Marginal Tax Rates

Notes: This figure shows average and marginal tax rates along the income distribution in the calibrated steady state. An income of 1 corresponds to median income in the calibration.

total income. The calibrated tax-and-transfer system is progressive in average rates because the income tax function is progressive and the transfer introduces additional progressivity into average rates of the overall system. Marginal tax rates are falling at the bottom of the income distribution, where transfers imply high marginal rates. Once transfers are phased out, marginal tax rates are also increasing because of the positive progressivity of the income tax function.

This leaves as the only remaining parameters the discount factor β, government debt D, labor disutility B, and exogenous government spending G. We set the former three parameters to match an interest rate of 2%, a government debt to output ratio of 60%, and an average labor supply of 0.3. Spending is implied by government budget clearing. The resulting spending to output ratio of around 14% is roughly in line with the data.

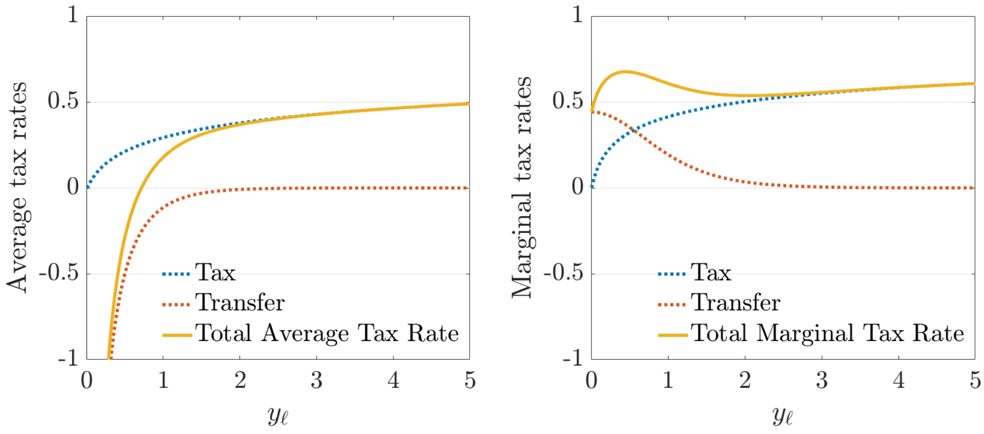


Figure 3.13 Optimal Tax-and-Transfer System: Average and Marginal Tax Rates

Notes: This figure shows optimal average and marginal tax rates along the income distribution. An income of 1 corresponds to median income in the calibration.

3.3.3 Optimal Tax-and-Transfer System

We use our calibrated model of the U.S. economy with realistic distributions of income, wealth, and income risk to compute the optimal tax-and-transfer system. In order to find the optimal system, we optimize on the fiscal system parameters λ, θ, and ξ. The level of the transfer m is residually determined from the government budget. The government can change the tax-and-transfer system once and for all, i.e. the three parameters of the fiscal system that we are choosing optimally are not allowed to vary over time.[4] However, we take into account the entire transition towards a new steady state when computing welfare. Along the transition, m varies to clear the government budget period by period.

The optimal tax-and-transfer system is much more redistributive than the system currently in place in the United States. Optimal transfers are large with an m of 0.46. They phase out slowly with $\xi = 1.94$. The large transfers are financed with moderately progressive taxes ($\theta = 0.17$). Average and marginal tax rates along the income distribution are shown in Figure 3.13. Again, the graph is for a household with zero capital income, with y_ℓ being normalized by median income. Average tax rates of the entire system are very progressive because the income tax function is progressive and the transfer is large. Marginal tax rates, however, are not monotonically increasing. Rather, they are high at low income levels because the large transfer is phased out. At high incomes marginal tax rates are increasing because the transfer has phased out completely and progressivity is solely determined by the tax function.

[4]Optimal time varying tax systems are investigated in Acikgöz, Hagedorn, Holter, and Wang (2018) and Dyrda and Pedroni (2020).

Table 3.4 Optimal Tax-and-Transfer System

Data	Q1	Q2	Q3	Q4	Q5
Tax rate	0%	10%	16%	20%	27%
Transfer rate	26%	3%	1%	0%	0%
Total avg rate	-26%	-7%	15%	20%	27%
Optimal	Q1	Q2	Q3	Q4	Q5
Tax rate	15%	21%	27%	31%	44%
Transfer rate	170%	58%	21%	6%	0%
Total avg rate	-155%	-37%	6%	25%	44%
Marginal rate	62%	66%	62%	53%	51%

Notes: This table shows average tax and transfer rates in the data and under the optimal tax-and-transfer system in the model for income quintiles. For the model, the table also shows marginal tax rates.

Table 3.4 shows the optimal average and marginal rates by income quintile. Overall, the tax-and-transfer system is very progressive in average rates. The bottom quintile's average tax rate is -155%, whereas the top income quintile faces an average tax rate of 44%. However, marginal tax rates of the entire system look very different. They are highest in the lowest income quintiles and lowest at the top. Thus, we recover the result from the theoretical part that it is optimal to disentangle progressivity in average and marginal tax rates. Because income and wealth are distributed very unequally a utilitarian planner wants to redistribute a lot. The efficient way of achieving this is to give large transfers, but keep marginal tax rates at the top at a relatively lower level.

We can also recover the second key insight from the theoretical model in the quantitative part: The larger transfers, the lower the optimal progressivity of the tax system. We illustrate this finding in Figure 3.14. In the figure we plot the optimal progressivity parameter θ as a function of the level of the transfer m, keeping the phase-out constant at $\xi = 2$. The figure shows that optimal progressivity is decreasing in the size of the transfer. Thus, this key insight survives in the more complicated model with capital, savings, and taking into account transitions.[5] Still, taxes are progressive, as opposed to the simple model. One key difference between the simple model and the quantitative model is that the productivity and income distribution has a Pareto tail. This is known to change the shape of the optimal marginal tax schedule at the top (see e.g. Mankiw, Weinzierl, and Yagan (2009) and Heathcote and Tsujiyama (2021)). With a lognormal income distribution, at a high income level there is only little mass above that income level, so that raising marginal taxes there, while distortive, raises only very

[5]Note that for this figure we optimize over θ and m, adjusting λ along the transition, leading to marginally different optimal combinations of parameters. This will be consistent in future versions.

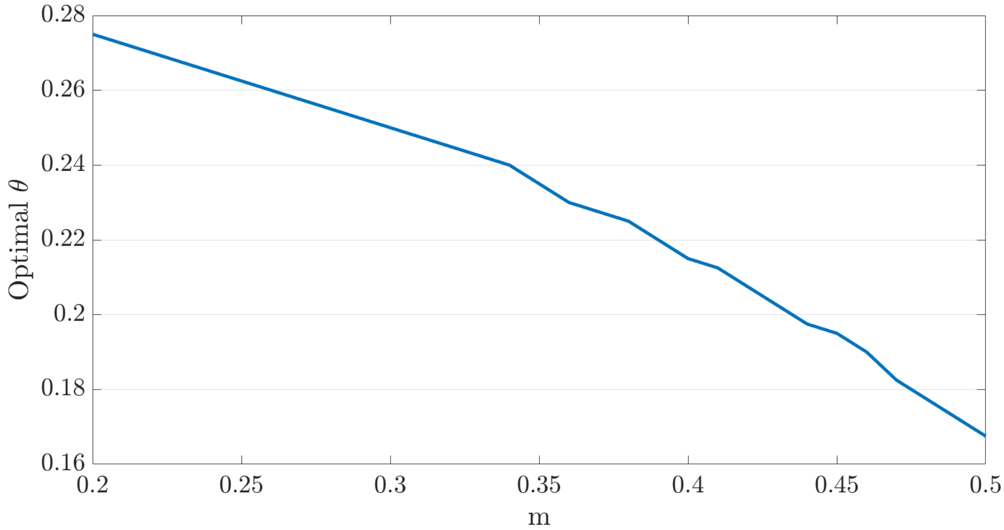


Figure 3.14 Quantitative Model: Relation between Transfers and Tax Progressivity

Notes: This table shows optimal progressivity of the income tax function θ as a function of the transfer level m, keeping the phase-out constant at $\xi = 2$. Along the transition the tax function level λ adjusts to clear the government budget.

little additional revenue to be redistributed to the poor. With a Pareto distribution, however, there is more mass at higher income levels such that higher marginal tax rates at high income levels become optimal because they can raise a lot of revenue. Given this and the large inequality in the U.S. the optimal system combines very generous transfers with moderately progressive taxes.

The optimal tax-and-transfer system does not just increase utilitarian welfare, but is also favored by a majority of households over the status quo. On aggregate, consumption equivalent welfare gains are very large with 9.64%. 79% of households would benefit. The largest welfare gains accrue to the poor, who benefit from the very generous transfers. Another group of households that gains is the group of asset rich households. With the more generous system in place, the capital stock along the transition towards the new steady state shrinks and asset rich households hold the assets that can be consumed along the transition. They also benefit from rising interest rates. There is, however, also a minority of households who prefer the status quo over moving to the new system: High productivity, low asset households would suffer from the reform because they face higher tax rates. The distribution of consumption equivalent welfare gains is shown in Figure 3.15.

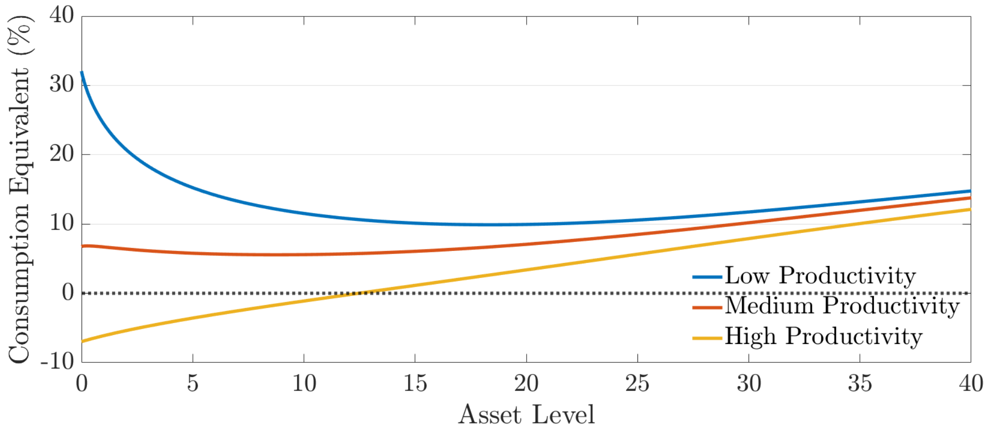


Figure 3.15 Optimal Tax-and-Transfer System: Consumption Equivalent Welfare Gains

Notes: This figure shows consumption equivalent welfare gains from a tax reform to the optimal system for different parts of the asset-productivity distribution.

3.3.4 Investigating the Results

How Important is Phasing-Out?

The optimal tax-and-transfer system features phasing-out transfers. We incorporate this feature into our tax function because it is a feature of the U.S. tax code and because it allows us to have non-monotonic marginal tax rates. In particular, it allows for a U-shaped marginal tax rates schedule that is often found to be optimal in the Mirrleesian optimal tax literature.

We now investigate whether it is a crucial feature to achieve the large welfare gains to allow for phasing-out transfers. For that purpose we compute the optimal system restricting ξ to zero. In that case, all households receive a lump-sum transfer or universal basic income. It is taxed away only through the income tax function but not phased out.

In Table 3.5 we compare the average and marginal tax rates of the whole system with phasing-out transfers to the system with lump-sum transfers. Having phasing-out transfers allows to have slightly larger transfers at the bottom, so that the average tax rate at the bottom is more negative with phasing-out transfers. However, overall both systems are very redistributive with strongly increasing average tax rates along the income distribution. The marginal tax rates schedules are, however, quite different. With lump-sum transfers the functional form assumptions do not allow for higher marginal tax rates at the bottom than at the top. Consequently, with the lump-sum the marginal tax rates schedule is rather flat. This contrasts with the lower marginal tax rates at the top with phasing-out transfers.

However, in welfare terms the optimal system restricted to lump-sum transfers comes relatively close to the optimal system with phasing-out transfers. The consumption equivalent gain is 9.43%,

Table 3.5 Optimal Phasing-Out Transfers vs. Lump-Sum Transfers

With phase-out	Q1	Q2	Q3	Q4	Q5
Total avg. rate	-155%	-37%	6%	25%	44%
Total marg. rate	62%	66%	62%	53%	51%
Lump-sum	Q1	Q2	Q3	Q4	Q5
Total avg. rate	-125%	-29%	4%	20%	45%
Total marg. rate	60%	61%	62%	63%	64%

Notes: This table compares optimal average and marginal tax rates of the entire tax-and-transfer system for two cases: phasing-out transfers and lump-sum transfers.

which is almost as large as with phasing-out transfers. The fraction of households preferring the system over the status quo is even slightly larger than in the previous case. However, it requires very high rates of the income tax function, which may still make it hard to implement.

How Important are Departures from Normality?

For the income process in our benchmark calibration we make two deviations from the standard assumption of normality. First, we add a Pareto tail to the productivity distribution. Second, we draw shocks from a mixture of normals, allowing us to match higher order moments of earnings risk. We now investigate how important these deviations are for the optimal tax-and-transfer system.

First, we remove the Pareto tail. We recalibrate the model to match the targets as in our benchmark calibration. Most importantly we need a different discount factor to match an interest rate of 2% because we are now missing the richest agents who contribute significantly to aggregate savings. In the optimal system, transfers at the bottom are lower with $m = 0.43$, but phase-out at a lower rate $\xi = 1.65$. Progressivity of the tax schedule is lower with $\theta = 0.09$. The lower progressivity is driven by the fact that there is less mass in the very right tail of the income distribution, which reduces the potential to "soak the rich" to finance generous transfers. Having high marginal tax rates at high income levels remains distortive but raises less revenue. Therefore, progressivity is lower optimally. Given that inequality is less pronounced transfers at the bottom of the income distribution are smaller. The average tax rates of the entire system are compared to our benchmark model in Table 3.6: They are higher at the bottom and lower at the top of the distribution.

Second, starting from the version of the model without the Pareto tail, instead of drawing shocks from a mixture of normals we rely on a simple AR(1) process for productivity. We choose the standard deviation of this process to be consistent with the standard deviation of the mixture. This implies that we are missing higher order moments of earnings risk. Even though with endogenous labor the earnings

Table 3.6 Optimal Tax-and-Transfer System with Normal Shocks

Total avg rate	Q1	Q2	Q3	Q4	Q5
Benchmark	-155%	-37%	6%	25%	44%
No Pareto tail	-131%	-26%	10%	28%	39%
AR(1)	-151%	-35%	5%	27%	41%

Notes: This table compares optimal average tax rates of the entire tax-and-transfer system for the benchmark economy, an economy witout the Pareto tail for the productivity distribution, and an economy without Pareto tail and with shocks drawn from a normal distribution instead of a mixture of normals.

growth distribution does not inherit the exact moments from the productivity process, the earnings growth distribution with this shock process is essentially symmetric and not leptokurtic. There are now many more households who experience medium sized productivity changes and many less with very small and very large changes.

The average tax rates of the optimal system with this shock structure are also shown in Table 3.6. Maybe surprisingly, the optimal system is more generous compared to the GMAR productivity process ($m = 0.45$, $\xi = 1.40$, $\theta = 0.08$). As there is no Pareto tail, progressivity of the income tax is mild. The transfer, however, is larger. This indicates that increasing the likelihood of medium sized earnings changes all the time outweighs the reduction in the probability of very large earnings changes.

How Important is Wealth Inequality?

In our benchmark calibration we match well the distributions of income and income risk. There is also significant wealth inequality; we miss, however, the extreme concentration of wealth at the very top of the distribution. To improve the fit of the model in this dimension we consider an extension with heterogeneous stochastic discount factors as in Krusell and Smith (1998). We allow for two different discount factors with transition probabilities chosen such that 10% of the population have the high discount factor. The process is very persistent. We compare the wealth distributions in the data, our benchmark calibration, and the extension with heterogeneous discount factors in Table 3.7. The extended model captures the extreme concentration of wealth well, with 69% of the wealth in the economy held by the top 10% of the distribution compared to 71% in the data.

We show the average tax rates under the optimal system in Table 3.8. With an m of 0.47 and a phase-out ξ of 0.5 the transfer is more generous than in our benchmark. Tax progressivity θ is lower, but tax levels are higher to finance the larger transfer. While the combination of instruments is slightly different than in the benchmark, overall average tax rates of the entire system are not too different.

Table 3.7 Wealth Distributions with Heterogeneous β

Net worth dist.	Q1	Q2	Q3	Q4	Q5	Top 10
Data	-1%	1%	3%	9%	88%	71%
Benchmark	0%	2%	8%	18%	72%	52%
Het. β	0%	1%	3%	11%	85%	69%

Notes: This table shows the distributions of net worth in the data, in the benchmark model, and in the extended model with heterogeneous stochastic discount factors.

Table 3.8 Optimal Tax-and-Transfer System with Heterogeneous β

Total avg. rate	Q1	Q2	Q3	Q4	Q5
Benchmark	-155%	-37%	6%	25%	44%
Het. β	-153%	-35%	1%	22%	47%

Notes: This table compares the optimal average tax rates of the entire tax-and-transfer system of the benchmark model and the extended model with heterogeneous stochastic discount factors.

How Large are Gains compared to Loglinear Function?

We have shown that while using phasing-out transfers is optimal, the largest share of the welfare gains can already be achieved by only using lump-sum transfers. We now investigate whether restricting the government to the loglinear tax function without even the lump-sum transfer also comes close to the optimal system. The optimal progressivity parameter with the loglinear tax function is 0.49, which also achieves more redistribution than the system currently in place. However, it enforces a tight link between marginal and average tax rates, so that to achieve this large degree of redistribution marginal tax rates at the top become very high. The consumption equivalent welfare gains from moving to this system are 5.08%. While this is large, it clearly falls short of the gains achieved with the more flexible tax functions.

How Important are Transitions?

As our baseline, we compute welfare after a change to the tax system taking into account the entire transition to the new steady state. This is conceptually the right measure because the distributions over assets differ potentially significantly between the calibrated initial steady state and the final steady state after a policy change. Transitions to new steady states are generally quite slow, so just comparing steady states can be severely misleading. This is the case in particular because the capital stock in a new steady state might be higher. It might be good in welfare terms to be in a steady state with a higher capital stock when not taking into account the transition during which this larger capital stock has to be accumulated.

With this caveat in mind we still compute the optimal steady state tax-and-transfer system. The optimal transfer is large with $m = 0.36$. It does not phase-out optimally and progressivity of the income tax is mild ($\theta = 0.03$). This reflects that having a larger capital stock is attractive when only comparing steady states: The system provides less redistribution than the optimal system with transitions, which encourages more precautionary savings. There is no phase-out because the phase-out also depends on capital income and imposing a higher marginal tax on capital is undesirable in this case. Progressivity is low to not discourage capital accumulation by the most productive too much. Then, under the optimal system the capital stock will be higher than in our benchmark, which makes it preferable to be in this steady state.

3.4 Conclusion

In this paper, we study the optimal design of the tax-and-transfer system. First, we establish in a simple analytical framework that there is a negative optimal relationship between the size of transfers and the progressivity embedded in the income tax code. This is the case for efficiency and redistribution reasons: When the government has to finance a large transfer it is optimal to incentivize individuals to work more through lower progressivity in marginal rates. Also, transfers compress the consumption distribution, lowering the desire to reduce inequality more through the income tax code.

In a quantitative model of the U.S. economy with realistic distributions of income, wealth, and income risk, we find that transfers should be more generous and taxes should be higher. However, taxes should not be more progressive. This combination of instruments allows to achieve a large reduction in post-tax-and-transfer inequality, while preserving efficiency with marginal progressivity lower than average progressivity. It is key for achieving large welfare gains to separate marginal and average progressivity in this way. Adding additional flexibility to the tax-and-transfer schedule by allowing transfers to phase-out, which allows for non-monotonic (U-shaped) marginal tax rates schedules, increases welfare further, but is quantitatively less important.

Chapter 4

Redistribution in Growing Economies[1]

Abstract We analyze the dynamics of the equity-efficiency trade-off along the growth path. To do so, we incorporate the optimal income taxation problem into a state-of-the-art multi-sector structural change general equilibrium model with non-homothetic preferences. We identify two key opposing forces. First, long-run productivity growth allows households to shift their consumption expenditures away from necessities. This implies a reduction in the dispersion of marginal utilities, and therefore calls for a welfare state that declines along the growth path. Yet, economic growth is also systematically associated with an increase in the skill premium, which raises inequality and the desire to redistribute. We quantitatively analyze these opposing forces for two countries: the U.S. from 1950 to 2010, and China from 1989 to 2009. Optimal redistribution decreases at early stages of development, as the role of non-homotheticities prevails. At later stages of development the rising income inequality dominates and the welfare state should become more generous.

4.1 Introduction

Fiscal redistribution is a central tool for governments to reduce poverty and inequality. A large literature has analyzed the optimal design of the welfare state, which is shaped by the classic trade-off between efficiency and redistribution: higher taxes allow for a more generous redistribution, but disincentivize labor supply. This literature has primarily focused on steady-states and business-cycle fluctuations in one-sector economies, trying to understand how the welfare state should look like in stationary economies and how it should adjust during expansions or recessions.

[1] This paper uses data from the U.S. Census and the American Community Survey provided by IPUMS USA (Ruggles, Flood, Goeken, Grover, Meyer, Pacas, and Sobek, 2020).

We adopt a different perspective and consider long-run productivity growth. Growth has potentially large and conflicting effects on poverty and inequality. On the one hand, as an economy becomes richer, absolute poverty tends to decrease;[2] necessities account for a decreasing share of aggregate consumption, as emphasized in the structural change literature. On the other hand, a growing economy typically reallocates towards high-skill intensive industries, which increases the demand for high-skilled workers. The skill premium rises, and thus inequality tends to increase.[3] Finally, growth may also change labor elasticities over time, altering the efficiency costs of taxation. Overall, long-run productivity growth generates non-trivial dynamics for the costs and needs for redistribution over time.

This paper analyzes how the welfare state should adjust over time in growing economies. To that end, we combine the workhorse models of public finance and macro development. We build a heterogeneous-agent variant of a state-of-the-art structural change general equilibrium model with non-homothetic CES preferences (Buera, Kaboski, Rogerson, and Vizcaino, 2021; Comin, Lashkari, and Mestieri, 2021). We incorporate the optimal income taxation problem into this framework to analyze the dynamics of the equity-efficiency trade-off due to aggregate growth and changes in the composition of consumption.

First, we study how the optimal fiscal plan changes with growth in a simplified partial equilibrium set-up. We characterize the fully optimal nonlinear tax schedule in the spirit of Mirrlees (1971) and show that, absent changes in prices and wages, the main channel at play with non-homothetic preferences is a reduction in the dispersion of marginal utilities. As a result, optimal redistribution decreases over time. We also document that the optimal tax schedule is well approximated by the parametric function developed in Ferriere, Grübener, Navarro, and Vardishvili (2021), which motivates us to analyze optimal nonlinear taxes using this parametric tax function in the general equilibrium environment.

We then turn to the general equilibrium model, to account for the fact that sectoral reallocation along the growth path and skill-biased technological change endogenously increase the skill premium. We calibrate our model to two countries: the U.S. from 1950 to 2010, and China from 1989 to 2009. We find that optimal redistribution decreases at early stages of development even when inequality rises, as the role of non-homotheticicities prevails. On the other hand, at later stages of development the rising skill premium and income inequality dominate, so that the optimal tax-and-transfer system becomes more redistributive.

We build a rich structural change model with three sectors, agriculture, manufacturing goods, and services, and households heterogeneous in their skill and productivity. The key ingredients of the

[2] See Ferreira and Ravallion (2011).

[3] This mechanism, which is referred to as "skill-biased structural change" by Buera, Kaboski, Rogerson, and Vizcaino (2021), is further amplified by skill-biased technical change.

model are (i) non-homothetic constant-elasticity-of-substitution preferences à la Comin, Lashkari, and Mestieri (2021) with sector-specific income elasticities; and (ii) differential skill intensities across sectors: Agriculture uses high-skilled labor relatively little, whereas services represent the most high-skill intensive sector. In poor countries, households spend a large share of their income on agricultural necessity goods. With aggregate growth and non-homothetic preferences, households endogenously shift their consumption away from agricultural goods, toward goods and eventually services. This pattern creates the typical hump-shaped profile for the manufacturing share in GDP and an ever increasing share of services in aggregate output. In addition to this demand-driven structural change, the model allows for heterogeneous growth rates of productivity across sectors and skill-biased technical change. In general equilibrium, the increasing demand for high-skilled workers will generate an endogenous increase in the skill premium.

We start with a simplified partial equilibrium setup to analyze the effect of growth on optimal redistribution in the presence of non-homothetic preferences. In the canonical optimal taxation problem with homothetic preferences aggregate growth is irrelevant for optimal redistribution; the only relevant statistic is relative inequality. With non-homothetic preferences, growth reduces dispersion in marginal utilities, and thus reduces the need for redistribution over time. In a low-productivity economy, poor households spend most of their income on agricultural necessity goods. As the economy grows, all households eventually decrease their share of spending on agricultural goods, reflecting the fact that primary needs are satisfied. Marginal utilities decrease, especially for the poorer. With non-homothetic preferences, growth also affects labor supply elasticities. However, we show that this effect is quantitatively smaller. We calibrate the partial equilibrium model to the United States in 1950. We pursue an inverse optimum approach and choose welfare weights such that the observed 1950 tax-and-transfer system is optimal given the chosen welfare weights. Then, we increase all incomes by a factor corresponding to GDP per capita growth until 2010, keeping relative inequality and relative prices constant. We show that because of the economy getting richer and absolute poverty concerns becoming less severe marginal tax rates fall across the entire income distribution. This translates into smaller transfers paid out to households, so that the welfare state is much less generous.

We then return to the general equilibrium model to analyze the dynamics of the welfare state when also accounting for price and wage changes. We calibrate the model to match salient features of structural change in the U.S. from 1950 to 2010 and in China from 1989 to 2009. In both countries over the respective time period the skill premium rose significantly. The model accounts for that with a combination of skill-biased structural change and skill-biased technical change.

In our calibrated model optimal redistribution should become more generous over time in the United States. However, this optimal evolution of the welfare state is non-monotonic. From 1950 to 1980, the welfare state should become less generous, as the rise in inequality through an increasing skill premium is relatively small and the effect of non-homotheticities is relatively large. From 1980 to 2010 there is a much larger increase in inequality and non-homotheticities play a smaller role because the economy is already larger to start with. Therefore, the rising inequality effect dominates and the government should redistribute more. We decompose the changes further and show that the welfare state should become less generous over time because of aggregate growth and because relative prices shift such that the consumption baskets of the poor become relatively cheaper. On the other hand, the welfare state should become more generous because the skill premium and the share of high-skilled individuals rises.

The effects of non-homotheticities are even more important for China. From 1989 to 2009 there is a large rise in the skill premium in China. On the other hand, there is also very strong growth in GDP per capita. The agriculture share was initially quite high and decreased substantially over that time period. Even with the large rise in inequality the welfare state should not become much more redistributive; over the first half of that period it should have even become much less generous because of the strong effect of non-homotheticities.

Related Literature. This paper relates to two large but separate literatures. A large literature studies optimal taxation in stationary environments. One strand of this literature following Mirrlees (1971) analyzes the unrestricted fully non-linear optimal tax schedule in usually relatively simple models (Diamond, 1998; Saez, 2001). Much of this strand of the literature focuses on static environments, while Farhi and Werning (2013) and Golosov, Troshkin, and Tsyvinski (2016) extend the approach to dynamic settings. Another strand of the optimal taxation literature following Ramsey (1927) exogenously restricts the tax instruments available to the government, but usually analyzes richer models (Bhandari, Evans, Golosov, and Sargent, 2017; Heathcote, Storesletten, and Violante, 2017; Werning, 2007). Relative to the optimal taxation literature, we embed the optimal tax problem into a state-of-the-art model of structural transformation and aggregate growth instead of considering a stationary environment.

The second large literature this paper relates to is the literature on structural transformation. This shift of the economy out of agriculture to manufacturing and increasingly services has long been considered to be one of the key features of growing economies (Kuznets, 1973). The literature usually considers two main drivers of the structural transformation. The first driver are income effects through non-homothetic preferences, as emphasized by Kongsamut, Rebelo, and Xie (2001). As households become richer, they demand more and more manufactured goods and services such that the economy shifts out of agriculture towards these sectors. The second key driver is differential productivity growth across sectors (Ngai and Pissarides, 2007). Both of these forces are considered to be important drivers of the structural transformation and are therefore both incorporated into state-of-the-art models of structural change (Duarte and Restuccia, 2010; Herrendorf, Rogerson, and Valentinyi, 2014), so we also allow for both of them in our framework. While much of this literature focuses on representative agent models, we build on Buera, Kaboski, Rogerson, and Vizcaino (2021) and Fang and Herrendorf (2021), who incorporate some limited household heterogeneity. Most of the literature uses Stone-Geary preferences to capture non-homotheticities (Geary, 1950). However, a number of recent papers have shown that other non-homothetic preferences fit some important features of the data better (Boppart, 2014; Święcki, 2017). In that respect, we use the non-homothetic CES preferences proposed for a model of structural change by Comin, Lashkari, and Mestieri (2021).

Besides these two large literatures, this paper shares the interest in designing the welfare state in growing economies with Song, Storesletten, Wang, and Zilibotti (2015). Their focus is on the design of the Chinese pension systems.

Roadmap. The paper proceeds as follows. Section 4.2 summarizes some empirical regularities through which growth might matter for optimal redistribution. In Section 4.3 we introduce the partial equilibrium setup, in which we incorporate aggregate growth and non-homothetic preferences into the optimal income taxation problem. In Section 4.4 we extend the framework to general equilibrium to also capture the dynamics of inequality. We calibrate the model to the United States and China and perform the optimal tax analysis in general equilibrium. Section 4.5 concludes.

4.2 Motivating Facts

Before going to the model, we describe a number of key observations about economic growth. We focus on observations that might matter for how much a government might want to redistribute.

Observation 1: Absolute poverty. Economic growth can be a powerful mechanism to alleviate absolute poverty. Ferreira and Ravallion (2011) show that it is a robust feature of long-run growth that it lifts people out of absolute poverty. As a particularly impressive example, consider the case of China. According to data provided by the World Bank, in 1990 around two thirds of the Chinese population had less than \$1.90 (at 2011 international prices) available per day. This share fell to almost zero in 2016.[4]

[4]The data is from the World Development Indicators.

This may be important for a government's taste for redistribution, as redistribution might provide the largest benefits if it is directed to the very poor.

Observation 2: Structural change. One of the most prominent features associated with aggregate growth is the structural transformation (Jones, 2016). The classic split of the economy is into three sectors, which are agriculture, goods/manufacturing, and services. It is a very robust regularity across countries that the nominal agriculture share is constantly declining, the manufacturing share is hump-shaped, and the services share is rising. In combination with the next two observations this may have important implications for redistribution.

Observation 3: Relative prices. A related empirical regularity is the evolution of relative prices. It is a robust feature of development that the relative price of services is rising with growth (Buera and Kaboski, 2012; Duarte and Restuccia, 2017). This may have important implications for redistribution, as households at different positions in the income distribution consume very different consumption baskets. Price changes of different consumption baskets associated with growth may affect the need for redistribution.

Observation 4: Skill intensity. The next related observation is that the service sector tends to be the most high-skill intensive in the sense that it uses a lot of high-skilled labor in production. The shift towards this sector may therefore drive up the demand and price of high-skilled labor, with important implications for inequality, as pointed out by Buera and Kaboski (2012) and Buera, Kaboski, Rogerson, and Vizcaino (2021).

Observation 5: Inequality. A final robust feature observed for a variety of countries is the rise of the skill premium. Wage inequality between college educated and non-college educated workers is increasing. This is well known for the U.S. (Katz and Murphy, 1992), but also true for other countries such as China (Fang and Herrendorf, 2021). Technical change is closely related to this phenomenon through the shift towards high-skill intensive sectors described above or skill-biased technical change.

4.3 Mirrleesian Setup in Partial Equilibrium

In this part of the paper we want to isolate and quantify the effect of growth on optimal redistribution when preferences are non-homothetic. For this purpose, we incorporate non-homothetic CES preferences à la Comin, Lashkari, and Mestieri (2021) into the workhorse model of optimal taxation following Mirrlees (1971), Diamond (1998), and Saez (2001).

4.3.1 Households

There is a continuum of heterogeneous households, who are characterized by their labor productivity θ. They choose labor supply n, so that their pre-tax labor income is given by $y = \theta n$. They consume goods from three sectors: agriculture, goods, and services. Households' preferences are defined by the following utility function over a consumption aggregator $C = \mathscr{C}(C_A, C_G, C_S)$, defined below, and labor supply n:

$$U(C,n) = \frac{C^{1-\gamma}}{1-\gamma} - B\frac{n^{1+\frac{1}{\varepsilon}}}{1+\frac{1}{\varepsilon}}, \tag{4.1}$$

where γ is the coefficient of relative risk aversion, ε is the Frisch elasticity of labor supply, and B is an additional labor disutility parameter.

Households are assumed to be hand-to-mouth. Therefore, they choose consumption and labor subject to their static budget constraint

$$p_A C_A + p_G C_G + p_S C_S = y - \mathscr{T}(y), \tag{4.2}$$

where p_j denotes the price of commodity j and $\mathscr{T}(y)$ is the nonlinear income tax schedule faced by the households.

For the consumption aggregator, we assume non-homothetic CES preferences following Comin, Lashkari, and Mestieri (2021).[5] It is standard in the literature on structural transformation to assume non-homothetic preferences. Indeed, non-homothetic preferences are frequently identified as one of the most important drivers of structural change (Kongsamut, Rebelo, and Xie, 2001). The most common specification of non-homotheticities are preferences of the Stone-Geary type (Herrendorf, Rogerson, and Valentinyi, 2014). Non-homothetic CES preferences share key properties with Stone-Geary preferences. Appropriately parameterized, agricultural goods are necessities, whereas services are luxuries. Stone-Geary preferences, however, have some disadvantages, which non-homothetic CES preferences overcome. First, the non-homotheticities vanish asymptotically meaning that as countries grow richer preferences behave as if they were homothetic. Second, they imply marginal propensities to consume out of a change in permanent income that are constant across income levels. Third, they

[5] To be precise, we use the formulation from Comin, Lashkari, and Mestieri (2017).

imply a functional relationship between income and price elasticities of demand. All these features are rejected by the data. None of the criticisms applies to non-homothetic CES preferences.

Utility is defined through the following constraint:

$$\sum_{j}^{J} (\Omega_j C^{\varepsilon_j})^{\frac{1}{\sigma}} C_j^{\frac{\sigma-1}{\sigma}} = 1. \tag{4.3}$$

The expenditure function implied by these preferences is

$$E(C;\mathbf{p}) = \left[\sum_{j}^{J} \Omega_j C^{\varepsilon_j} p_j^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \tag{4.4}$$

and the Hicksian demand function can be written as

$$C_j = \Omega_j \left(\frac{p_j}{E}\right)^{-\sigma} C^{\varepsilon_j}. \tag{4.5}$$

With these preferences, σ is the elasticity of substitution between goods. ε_j governs the income elasticity of demand. C is a consumption index, whereas C_j denotes the consumption of individual commodities.[6]

At first sight, when looking at the definition in equation 4.3, these preferences may seem hard to work with. They yield, however, a quite tractable demand system. This can be seen from the Hicksian demand function in equation 4.5. It can be easily seen from this equation that a good's share in expenditure will go up in income if ε_j is large and that the expenditure share will go down if ε_j is small. Hence, for a good to be a luxury, it has to have a large ε_j, whereas a low ε_j characterizes a necessity. Comin, Lashkari, and Mestieri (2021) estimate the preferences using micro data for countries at different stages of development and find stable parameters with the ε_j being largest for services and lowest for agriculture, with goods having an income elasticity between the two other sectors.

4.3.2 Government

The government chooses a fully unrestricted nonlinear tax schedule as in Mirrlees (1971), Diamond (1998), and Saez (2001). To define the government problem, we first write the household maximization problem:

$$\begin{aligned} V(\theta;\mathscr{T}(\cdot)) \equiv \max_{C,n}\ & U(C,n) \ \text{ s.t.} \\ p_A C_A + p_G C_G + p_S C_S &= n\theta - \mathscr{T}(n\theta). \end{aligned} \tag{4.6}$$

[6] We derive the expenditure function and the Hicksian demand function in Appendix D.1.

Households choose consumption C, defined by the non-homothetic CES consumption aggregator, and labor n to maximize their utility subject to their static budget constraint: Spending on the three commodities has to equal after-tax income. We denote by $V(\theta;\mathscr{T}(\cdot))$ the household value obtained by the optimal choice given the nonlinear tax schedule and by $n(\theta;\mathscr{T}(\cdot))$ the labor policy function.

Let $\{w(\theta)\}$ denote Pareto weights and G an exogenous spending requirement. In partial equilibrium we do not have to specify which commodities the government consumes. Then, we can write the government's maximization problem as

$$\begin{aligned} &\max_{\mathscr{T}(\cdot)} \int_{\underline{\theta}}^{\overline{\theta}} V(\theta;\mathscr{T}(\cdot))w(\theta)f(\theta)d\theta \ \text{ s.t.} \\ &[\lambda]: \int_{\underline{\theta}}^{\overline{\theta}} \mathscr{T}(n(\theta;\mathscr{T}(\cdot))\theta)f(\theta)d\theta \geq G. \end{aligned} \tag{4.7}$$

The government chooses a nonlinear tax schedule to maximize the integral over household utilities, weighted with the Pareto weights and the mass of households of a given productivity $f(\theta)$. The government is constrained by its budget constraint, stating that the revenues from the income tax have to cover the exogenous spending requirement. We denote by λ the multiplier on the government budget constraint, which is the marginal value of public funds.

4.3.3 Optimal Tax Formula

The optimal tax formula can be derived using standard techniques. Specifically, we use a perturbation approach following Saez (2001) to derive the optimal tax formula. We relegate the derivation to the appendix. Optimal marginal tax rates for a certain skill level θ^* are given by

$$\forall\, \theta^*: \ \frac{\mathscr{T}'(y(\theta^*))}{1-\mathscr{T}'(y(\theta^*))} = \left(1+\frac{1}{\varepsilon}\right)\left(1-\underset{\theta>\theta^*}{\mathbb{E}}\left[\frac{u'(\theta)}{E'(\theta)}\frac{w(\theta)}{\lambda} - \eta(\theta)\mathscr{T}'(y(\theta))\right]\right)\frac{1-F(\theta^*)}{f(\theta^*)\theta^*}.$$

This formula describes the standard efficiency-redistribution trade-off. The first term, $\left(1+\frac{1}{\varepsilon}\right)$, captures efficiency concerns. This tax formula is an inverse elasticity rule: The larger labor supply elasticities, the stronger individuals respond to higher taxes, and the lower marginal tax rates should be optimally. The term $\frac{u'(\theta)}{E'(\theta)}\frac{w(\theta)}{\lambda}$ accounts for redistributive concerns. If individuals with a higher skill than θ^* have very high incomes, their marginal utilities will be low, making this term small. Given that the term is subtracted, this means that this will be a force towards higher marginal tax rates. With high income inequality, implying low marginal utilities at the top, it is optimal to have high marginal tax rates because this raises large amounts of revenue from the top of the distribution to be redistributed at the bottom. $\eta(\theta)$ captures income effects. When marginal taxes at θ go up, this lowers incomes of everybody earning more. With leisure being a normal good, this will increase their labor supply. Hence, income effects are a force for higher marginal tax rates. Finally, the shape of the skill distribution matters. With a lot of mass above a certain skill level, there are many people whose average taxes are raised by an increase of marginal taxes at θ^*. This is a reason for higher marginal tax rates. However, if there is a lot of mass at this skill level, then distortions will be more important, so that lower taxes will be optimal.

Our description of the forces determining optimal taxes so far would apply equally to the case with homothetic preferences. They key difference applies to the term accounting for redistributive concerns $\frac{u'(\theta)}{E'(\theta)}\frac{w(\theta)}{\lambda}$. With homothetic preferences, $E'(\theta)$ is always equal to one for every skill level and therefore also independent of economic growth. However, with non-homothetic CES preferences this term causes redistributive concerns to become smaller as an economy becomes richer. Keeping relative inequality constant, increasing the incomes of all agents reduces the benefits from distribution. This is how the shift away from absolute poverty and the consumption of necessities enters the optimal tax formula.

Note that also the income effect term is affected by the presence of non-homotheticities. We derive the formula for the income effect in the appendix. We show in the next part, however, that the key change with non-homothetic preferences is to the taste for redistribution.

4.3.4 Calibration

We now evaluate the quantitative significance of growth in combination with non-homothetic preferences by computing the optimal income tax schedules in an economy that is calibrated to the U.S. in 1950 and for a counterfactual economy, in which every individual is richer by a factor corresponding to U.S. real GDP per capita growth from 1950 to 2010. For this exercise, we keep relative inequality constant.

For the calibration, we assume that the income distribution follows a log-normal distribution with a Pareto tail. To discipline the income distribution we use data from the 1950 U.S. Census (Ruggles, Flood, Goeken, Grover, Meyer, Pacas, and Sobek, 2020). As income measure we use wage and salary income. Because incomes are reported only in bins we refrain from estimating the parameters of the distributions directly. Rather, we set them such that the implied income distribution is consistent with the income distribution in the data. We assume that the income distribution for the majority of the population follows a log-normal distribution and only adjust the incomes of the top 5% such that they follow a Pareto distribution. We let the Pareto parameter decline linearly to ensure a smooth hazard ratio $\frac{1-H(y)}{h(y)}$, as in Sachs, Tsyvinski, and Werquin (2020). The income distributions in the data and in the model are shown in Table 4.1.

We fix a number of preferences parameters exogenously. We set the coefficient of relative risk aversion γ equal to two and the Frisch elasticity of labor supply ε equal to 0.2. We set the non-

Table 4.1 1950 U.S. Income Distribution

Decile	Data	Model
1	0.96%	2.47%
2	2.92%	4.24%
3	4.90%	5.07%
4	6.75%	7.13%
5	8.34%	7.41%
6	10.00%	9.67%
7	11.69%	10.46%
8	13.33%	13.25%
9	15.68%	15.44%
10	25.43%	24.85%

Notes: This table compares the income distributions for the U.S. in 1950 in the data and in the model.

homothetic CES parameters $\sigma = 0.3$, $\varepsilon_A = 0.1$, $\varepsilon_G = 1.0$, and $\varepsilon_S = 1.8$, in line with the estimates of Comin, Lashkari, and Mestieri (2021). We use these parameter values also in the general equilibrium model, so these parameters are also summarized in Table 4.3. In partial equilibrium, we also have to set the prices of the three commodities exogenously. Prices and the Ω_j parameters of the non-homothetic CES preferences cannot be distinguished from each other. They are jointly set such that we match the nominal sector shares of the U.S. in 1950, with roughly 60% of the economy in the services sector, 34% in the goods sector, and 6% in the agricultural sector. Because we match these targets exactly in our general equilibrium model, we incorporate the prices and preference parameters from the GE calibration into the PE environment. This gives us a very close match for the sector shares in PE.

We calibrate taxes and spending to be consistent with taxes and spending of the U.S. federal government in 1950. We assume that the government raises revenues using a parametric tax function. The tax payment of an individual earning income y is given in equation 4.8.

$$\mathscr{T}(y) = \exp\left[\log(\lambda)\, y^{-\tau}\right] y - T. \tag{4.8}$$

Individuals pay taxes on their income, where tax progressivity is determined by the parameter τ and the level of tax rates is determined by the parameter λ. Also, households receive a lump-sum transfer T. The level of the lump sum transfer is set to match government spending on income security (roughly 1% of GDP in 1950). Exogenous government spending G is set to account for all other federal government spending (roughly 14% of GDP). The parameters of the tax function are set to match average marginal tax rates along the income distribution.[7]

Finally, we back out the underlying skill distribution from the income distribution given all other parameters using the households' first order conditions, as in Saez (2001). For the optimal taxation problem, we follow an inverse optimum approach. That is, we compute Pareto weights such that the calibrated 1950 tax schedule is optimal given these weights (Bourguignon and Spadaro, 2012; Christiansen, 1977; Hendren, 2020; Lockwood and Weinzierl, 2016).

For our counterfactual 2010 economy, we scale down all prices by the same magnitude, equivalent to a proportional increase in income for everybody. We choose the magnitude to be the increase in GDP per capita from 1950 to 2010. We recalibrate taxes and spending to be consistent with the distribution of average marginal tax rates and government spending in the U.S. in 2010. Economic growth in combination with non-homothetic preferences leads to a shift in the consumption allocation across the different goods: Richer households spend less on agriculture and goods and more on services. The agriculture share falls from 6% to 2.5%; the goods share declines from 34% to 25%; and the services share rises from 60% to 73%. Note that this underestimates the structural change that we observe in the United States in the data. The reason is that we are looking at nominal sector shares but keep relative prices constant. However, it is well known that the relative price of services is rising with development ("Baumol's cost disease", Baumol (1967)). In the general equilibrium model we capture this force and thereby are able to match nominal sector shares. In this partial equilibrium framework we only capture a real shift towards services consumption and away from necessities, so that we do not account for the entire increase in the nominal service share.

Note that while relative inequality in incomes does not change by construction, the underlying inequality in the skill distribution does change. We are going to revisit the importance of that change at the end of the results section.

4.3.5 Optimal Taxes in Partial Equilibrium

Figure 4.1 depicts the optimal marginal tax rates in the two economies. The 1950 optimal marginal tax rates schedule is the calibrated tax schedule by virtue of the inverse optimum approach. Marginal tax rates are increasing along the income distribution. In addition to financing the exogenous government spending, the government uses the tax revenues to give a lump sum transfer of around 1.1% of GDP.

[7] This calibration of taxes and spending is similar to the general equilibrium model. We discuss data and targets in more detail there.

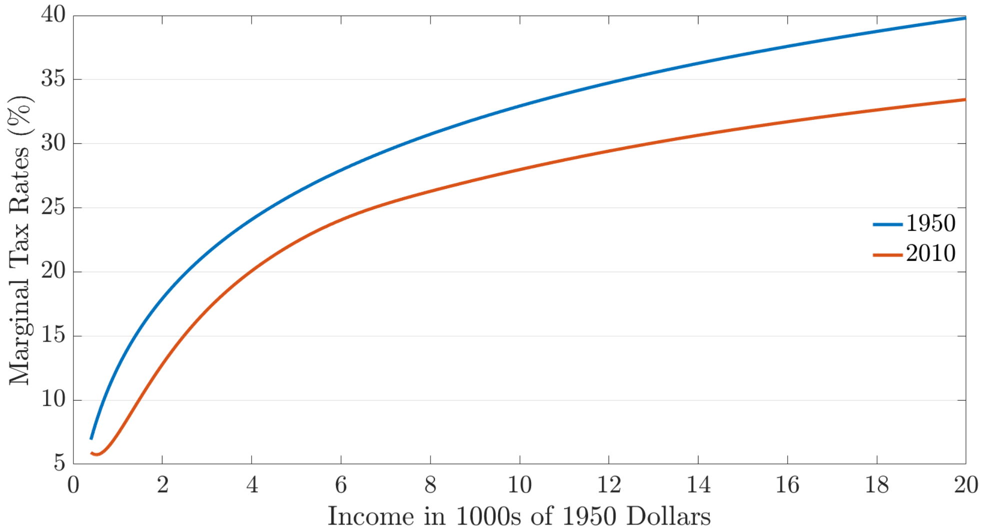


Figure 4.1 Optimal Marginal Tax Rates

Notes: This figure compares optimal marginal tax rates in the calibrated 1950 economy and the counterfactual 2010 economy, which is characterized by higher income levels.

This lowers average tax rates of the entire tax-and-transfer system to zero at the bottom, which is shown in Figure 4.2.

The two figures also show the optimal marginal and average tax schedules for the counterfactual 2010 economy. Optimal marginal rates are lower across the entire income distribution. Quantitatively, the drop is roughly 5% at most income levels, with a slightly smaller difference at the bottom of the distribution. These lower tax rates are not sufficient to finance the entire exogenous government spending, so that instead of a lump sum transfer there is a lump sum tax of 2% of GDP. This optimal system then implies very different average tax rates. The lump sum tax matters a lot for average tax rates at the bottom of the distribution. The poorest individuals now face much higher average tax rates. By contrast, average tax rates are much lower for high earners because the lump sum tax matters less for them and the marginal tax rates schedule is shifted downwards.

Hence, with non-homothetic preferences aggregate growth is associated with less redistribution. Absolute poverty is less of a concern and the consumption baskets also of the poorest contain more of the relative luxury good services and less of the necessity agriculture. We now show that it is indeed this declining taste for redistribution that accounts for most of the change to the optimal marginal tax rates and not the changing skill distribution or income effects.

Figure 4.3 shows again the marginal tax rates schedule for 1950 and 2010. Additionally, we consider two intermediate cases. First, we compute optimal rates for a scenario in which prices are at the 2010

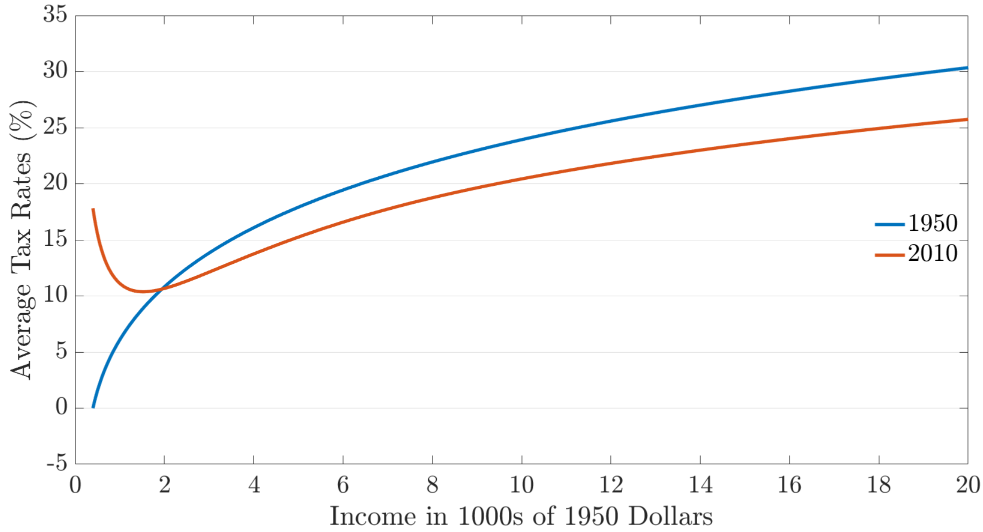


Figure 4.2 Optimal Average Tax Rates

Notes: This figure compares optimal average tax rates, implied by the entire tax-and-transfer system including the lump sum grant, in the calibrated 1950 economy and the counterfactual 2010 economy, which is characterized by higher income levels.

level, i.e. aggregate growth has happened. However, we keep labor supply and the skill distribution fixed such that not just income inequality but also underlying skill inequality is the same as in 1950. Note that because of the non-homotheticities labor supply is higher across the entire income distribution in 1950 compared to 2010. This effect is strongest for the lowest incomes: Because they are very poor in absolute terms in 1950, their labor supply is very high. The poorest in 2010 are much richer in absolute terms, even if their relative income share is the same. Therefore, the reduction in labor supply is largest at the bottom of the income distribution. To keep the income distribution the same, as imposed by the calibration strategy, we need that the skill distribution also changes most at the bottom of the distribution. All skill levels are lower in 1950, but the change is largest at the bottom. Therefore, if we do a counterfactual where we impose inequality in skills to be as in 1950, with income inequality being the same anyways, but with lower prices because of aggregate growth as in 2010, we have optimally higher marginal tax rates compared to 2010, bringing the tax rates schedule closer to the 1950 schedule. This is shown in Figure 4.3 in the case "2010 with 1950 inequality".

We consider a second intermediate case in which we also fix the income effects at the 1950 level. Income effects are larger in 1950 than in 2010, as on average poorer individuals respond more strongly to receiving unearned income. As explained above, larger income effects are a force for higher marginal tax rates. Increasing marginal tax rates at some point in the distribution is more beneficial as individuals earning higher incomes will work more because of the loss through higher average taxes. Hence, the

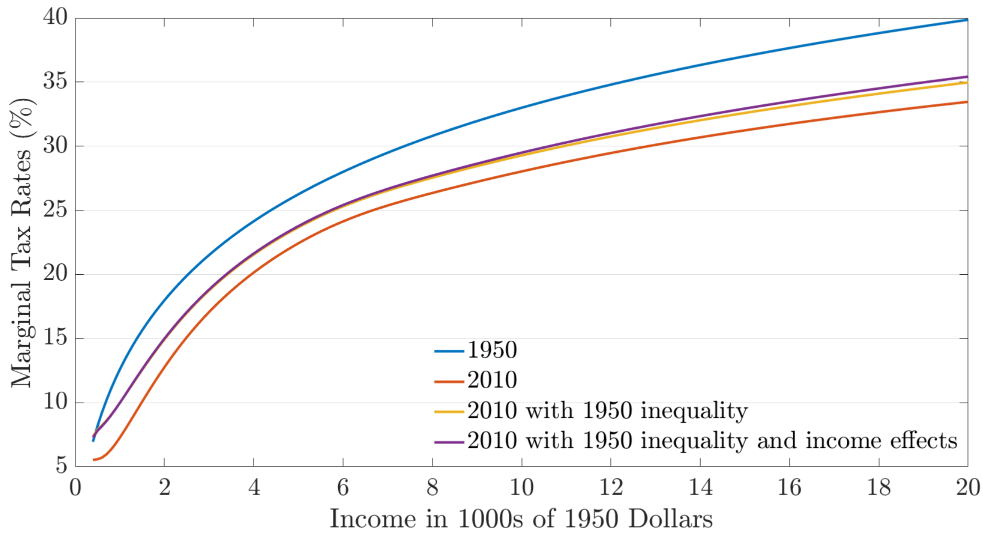


Figure 4.3 Optimal Marginal Tax Rates: Decomposition

Notes: This figure compares optimal marginal tax rates, implied by the entire tax-and-transfer system including the lump sum grant, in the calibrated 1950 economy and the counterfactual 2010 economy, which is characterized by higher income levels. It additionally compares two intermediate cases "2010 with 1950 inequality" and "2010 with 1950 inequality and income effects".

"2010 with 1950 inequality and income effects" schedule brings us even closer to the 1950 optimum. However, quantitatively these two changes account only for a small part of the change in the optimal tax-and-transfer systems between 1950 and 2010. Therefore, we conclude that the most important implication of non-homothetic preferences in combination with aggregate growth is the change to the taste for redistribution.

4.4 General Equilibrium

We now turn to the general equilibrium model, in which in addition to aggregate growth we account for changes to relative inequality and relative prices.

4.4.1 Households

In the general equilibrium model, household heterogeneity takes two dimensions. Households are characterized by their skill level, which can be high or low. Firms pay wages by skill level per efficiency unit of labor. Within skill, households differ in their productivity. Denote the wage a household receives with w and the productivity within skill with θ. Then, the household problem can be written as

$$
\begin{aligned}
&\max_{C,n} \frac{C^{1-\gamma}}{1-\gamma} - B\frac{n^{1+\varphi}}{1+\varphi} \\
&\text{s.t. } E(C) = w\theta n \left[1 - \exp\left[\log(\lambda)(w\theta n)^{-\tau}\right]\right] + T.
\end{aligned}
\tag{4.9}
$$

C is the non-homothetic CES consumption aggregator and n is labor supply. The budget constraint of the household equalizes consumption expenditure and after-tax-and-transfer income: λ is the level parameter of the parametric tax function, τ governs progressivity, and T is a lump sum transfer.

The first order condition of the household with respect to consumption reads

$$
C^{-\gamma} = \chi E'(C), \tag{4.10}
$$

where χ is the multiplier on the budget constraint. The first order condition with respect to labor is

$$
\begin{aligned}
Bn^{\varphi} &= \chi\Big\{w\theta\left[1 - \exp\left[\log(\lambda)(w\theta n)^{-\tau}\right]\right] \\
&+ w\theta n\left(-\exp\left[\log(\lambda)(w\theta n)^{-\tau}\right]\right)\log(\lambda)(-\tau)(w\theta n)^{-\tau-1}w\theta\Big\} \\
&= \chi\Big\{w\theta\left[1 - \exp\left[\log(\lambda)(w\theta n)^{-\tau}\right]\right] \\
&+ (w\theta)^{1-\tau}\exp\left[\log(\lambda)(w\theta n)^{-\tau}\right]\log(\lambda)\tau n^{-\tau}\Big\}.
\end{aligned}
\tag{4.11}
$$

We can combine these two first order conditions to

$$
0 = Bn^{\varphi} - \frac{C^{-\gamma}}{E'(C)}\left[w\theta\left[1 - \exp\left[\log(\lambda)(w\theta n)^{-\tau}\right]\right] + (w\theta)^{1-\tau}\exp\left[\log(\lambda)(w\theta n)^{-\tau}\right]\log(\lambda)\tau n^{-\tau}\right]. \tag{4.12}
$$

4.4.2 Production

The production side of the economy closely follows Buera, Kaboski, Rogerson, and Vizcaino (2021). The production function is given by

$$
Y_{jt} = A_{jt}\left[\alpha_{jt}H_{jt}^{\frac{\rho-1}{\rho}} + (1-\alpha_{jt})L_{jt}^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}}, \tag{4.13}
$$

where the subscript j denotes the sector, with $j \in \{A, G, S\}$ for agriculture, goods, and manufacturing and t denotes the time period. Y_j is sectoral output, which is produced using high-skilled and low-skilled labor. H_j and L_j are efficiency units of high-skilled and low-skilled labor, respectively. The parameters A_{jt} denote skill-neutral technology parameters, and α_{jt} denote skill intensities. These parameters are sector-specific and time-varying. The elasticity of substitution in production ρ is constant across time and sectors.

Firms maximize profits under perfect competition. The firm problem is static, so we omit the time subscript for notational convenience. The firm problem then reads:

$$\max_{H_j,L_j} p_j A_j \left[\alpha_j H_j^{\frac{\rho-1}{\rho}} + (1-\alpha_j) L_j^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}} - w_H H_j - L_j. \tag{4.14}$$

p_j denotes the price of commodity j. w_H is the wage paid per efficiency unit of high-skilled labor. The wage paid to low-skilled labor w_L is normalized to 1, so that w_H can be interpreted as the skill premium.

The solution of the firm problem implies that given the skill premium all prices are pinned down by parameters:

$$p_j = \frac{1}{A_j}\left[\frac{\alpha_j^{\rho}}{w_H^{\rho-1}} + (1-\alpha_j)^{\rho}\right]^{\frac{1}{1-\rho}}. \tag{4.15}$$

4.4.3 Government

The government raises income taxes to finance a lump sum transfer and exogenous government spending. The tax payment is given by the parametric tax function

$$\mathscr{T}(y) = \exp\left[\log(\lambda)\, y^{-\tau}\right] y - T, \tag{4.16}$$

where $\exp\left[\log(\lambda)\, y^{-\tau}\right]$ is the average tax rate applied to income $y = w\theta n$.

In addition to the lump sum transfer the government finances exogenous government spending in all sectors: G_a, G_g, G_s. The government budget constraint hence reads

$$\sum_{j=1}^{J} p_j G_j + T = \sum_{ik} \pi_{ik} w_i \theta_{ik} n_{ik} \exp\left[\log(\lambda)\,(w_i\theta_{ik}n_{ik})^{-\tau}\right], \tag{4.17}$$

where i denotes worker skill and k refers to within-skill productivity. π_{ik} is the mass of workers with skill i and within-skill productivity k.

4.4.4 Equilibrium

In equilibrium markets for high-skilled and low-skilled labor and markets for agriculture, goods, and services have to clear. From the firm problem, we can derive demand for high skilled labor as

$$H_j = \frac{Y_j}{A_j}\left(\frac{\alpha_j p_j A_j}{w_H}\right)^{\rho}. \tag{4.18}$$

Define also total consumption in each sector, consisting of private and government consumption, as

$$D_j = G_j + \sum_{ik} C_{jik}\pi_{ik}. \tag{4.19}$$

Imposing goods market clearing we can rewrite equation (4.18) as

$$H_j = \frac{D_j}{A_j}\left(\frac{\alpha_j p_j A_j}{w_H}\right)^{\rho}.$$

Then, labor market clearing in the market for high skilled labor requires

$$\sum_{Hk} \pi_{Hk} n_{Hk} \theta_{Hk} = \sum_{j=1}^{J} \frac{D_j}{A_j}\left(\frac{\alpha_j p_j A_j}{w_H}\right)^{\rho}. \tag{4.20}$$

This condition already imposes goods market clearing. Therefore, the only market left is the market for low-skilled labor, which clears by Walras' law if equation (4.20) holds.

The equilibrium definition is standard. The equilibrium consists of a set of prices and wages, given a tax-and-transfer system, such that households solve their utility maximization problem, firms solve their profit maximization problem, three goods and two labor markets clear, and the government budget is balanced. Before turning to the optimal tax problem in this economy, we discuss the calibration in the next section.

4.4.5 Calibration for the U.S.

Our first quantitative application of the model is to the United States. We calibrate the model to three years: 1950, 1980, and 2010. We choose parameters to be consistent with the two key determinants of optimal redistribution that we highlight: absolute poverty concerns and the level of the economy (aggregate growth, structural change) and relative inequality (changing relative prices, changing skill premium). We now describe our calibration strategy in detail, starting with a summary of the parameters to be chosen.

The first set of parameters we have to choose concerns demographics. In each period there is a share f_H of high-skilled individuals. Additionally, we have to parameterize the distribution of efficiency units within skill. We opt to keep within type inequality constant over time. While especially since the 1980s there also has been a rise of incomes of the very top earners relative to other highly educated individuals, we abstract from this change in order to not introduce more exogenous changes into the model. Another set of parameters that is kept constant over time is the set of all preference parameters. We need to choose the non-homothetic CES parameters $\sigma, \varepsilon_A, \varepsilon_G, \varepsilon_S, \Omega_A, \Omega_G, \Omega_S$ and the other preference parameters γ, φ, B. On the production side we need to set the elasticity of substitution in production ρ, which is constant over time, and the time-varying skill neutral technology parameters A_A, A_G, A_S as well as the time-varying skill intensities $\alpha_A, \alpha_G, \alpha_S$. Finally, we have to choose three parameters of the tax-and-transfer system λ, τ, T in every period.

We first fix a number of parameters exogenously. We set the non-homothetic CES parameters in line with the estimates of Comin, Lashkari, and Mestieri (2021) to $\varepsilon_A = 0.1$, $\varepsilon_G = 1.0$, $\varepsilon_S = 1.8$, and $\sigma = 0.3$. These preference parameters imply that agricultural goods are necessities, while services are luxuries. We set the coefficient of relative risk aversion γ to two and the Frisch elasticity of labor supply to 0.2, in line with micro estimates. We also exogenously fix the elasticity of substitution in production ρ at 1.42, as in Buera, Kaboski, Rogerson, and Vizcaino (2018). For the within type heterogeneity we distinguish between ten groups per skill level, accounting for 10% of the total mass per skill each. We set the efficiency units for each of these deciles based on within skill wage differences from U.S. Census data. Finally, we impose a few normalizations. We set Ω_G to one. We also fix A_S to one in 1980 and pick the productivity levels in agriculture and goods such that all relative prices are one in 1980.

We continue with calibrating the 1980 economy as our base year. Assuming that the skill premium is matched, it is possible to express the three skill intensities as functions of the share of income going to high-skilled individuals in each sector. We compute this data moment from the 1950 Census 1% Sample, the 1980 Census 5% sample, and the 2010 American Community Survey (Ruggles, Flood, Goeken, Grover, Meyer, Pacas, and Sobek, 2020). In the data, we consider an individual as high skilled if the person has at least a four year college degree. Everybody with less than a four year degree is considered low skilled. We restrict our sample to individuals who are between 25 and 60 years old. We use Census Bureau industrial classifications to group individuals into the three broad sectors of our model. As income measure we use total pre-tax wage and salary income. Income in the data is top-coded. We apply a simple adjustment to account for top-coding by multiplying all incomes at the top-coding threshold with 1.5. The high-skilled income shares by sector for the three years are reported in Table 4.2, which includes all the data moments targeted in our U.S. calibration. Two important

Table 4.2 Moments for the U.S. Calibration

Moment	**1950**	**1980**	**2010**
Skill premium	1.44	1.54	2.06
High-skill inc. share agriculture	5.22%	20.26%	23.62%
High-skill inc. share manufacturing	8.27%	19.96%	39.60%
High-skill inc. share services	16.46%	37.14%	57.67%
Service share	-	69.59%	-
Manufacturing share	-	28.04%	-
Real GDP per capita	0.50	1.00	1.70
Rel. price agriculture to manufacturing	1.88	1.00	0.55
Rel. price services to manufacturing	0.94	1.00	1.49
G/Y	14.00%	14.00%	14.00%
T/Y	1.12%	2.97%	3.61%
AMTR Top 10%	27.54%	46.03%	32.68%
AMTR Bottom 90%	14.94%	36.30%	27.95%

Notes: This table summarizes the moments for the U.S. calibration. Real GDP per capita and relative prices are normalized to 1 in 1980.

patterns emerge from this data. First, services is generally the high-skill intensive sector, with a larger share of income going to high-skilled individuals than in the other two sectors. Hence, as an economy grows richer and households shift their consumption towards services, this will drive up the demand for high-skilled labor and thereby put upward pressure on the skill premium. This is what Buera, Kaboski, Rogerson, and Vizcaino (2021) refer to as skill-biased structural change. The second key pattern is that the high-skilled income share goes up over time in each sector. This translates into rising skill intensities, which is commonly referred to as skill-biased technical change.

The remaining parameters that are calibrated internally are the non-homothetic CES parameters Ω_A and Ω_S, the high-skilled population share f_H, the labor disutility parameter B, and all the government parameters $\lambda, \tau, T, G_A, G_G, G_S$. They are calibrated jointly; here we discuss which data moments are most closely related to each of these targets. Ω_A and Ω_S are set to match the 1980 sector shares of goods and services. We compute these data moments from National Income and Product Accounts (NIPA), based on value added by industry. To smooth fluctuations we take five year averages around the respective years. The high-skilled population share is closely related to the skill premium, which we also compute from the Census data. The skill premium is defined as the ratio of average weekly earnings of high-skilled individuals divided by average weekly earnings of low-skilled individuals. We compute weekly earnings as annual wage and salary income divided by the number of weeks in employment. The labor disutility parameter B is set such that average labor supply of the high-skilled is equal to 0.33.

This leaves the government parameters to be calibrated. We discipline government spending using historical tables provided by the Office of Management and Budget. First, we discipline the size of the lump-sum transfer by matching spending on income security programs. This has increased over time, from 1.12% in 1950 to 3.61% in 2010. Again, these numbers are averaged over five year windows. We consider every other government expenditure as part of the exogenous spending requirement. Total expenditures have risen quite significantly over time. However, one aspect to be taken into account that is not modeled is that a sizeable part of the additional spending has been debt financed. This is the case for 1980 and even more so for 2010. When accounting for debt the financing requirement through taxes was relatively stable across the three years. Therefore, we choose to keep a constant spending to output ratio in the calibration. This has the advantage that our results are not affected by changing spending requirements over time. This could have sizeable effects on desired tax progressivity (Ferriere, Grübener, Navarro, and Vardishvili, 2021; Heathcote and Tsujiyama, 2021; Heathcote, Storesletten, and Violante, 2017), which we can abstract from by making this choice. In our three sector model we also have to take a stand on how government consumption is allocated across the three sectors. As our baseline assumption, we choose to set government spending as a proportional share of private consumption in each sector. Finally, to discipline the progressivity of the tax system we rely on average marginal tax rates estimated by Mertens and Montiel Olea (2018). The tax progressivity parameter τ is closely related to the difference between the average marginal tax rate (AMTR) faced by the top 10% of the income distribution compared to the AMTR of the bottom 90%. The remaining condition for the last government parameter λ is that the government budget has to clear.

Having calibrated the base year, we have to set the remaining parameters for the 1950 and 2010 economies. The skill intensities can be determined following the same approach as for the base year 1980 before calibrating the remaining parameters internally. Specifically, we need to determine skill-neutral technology parameters A_A, A_G, A_S, high-skilled population shares f_H, and all the government parameters. We discipline the skill-neutral technology parameters by matching growth rates in real GDP per capita and changes in relative sectoral prices. The data for sectoral prices is from NIPA, as for the sector shares. The skill-neutral technology parameters are rising over time, capturing aggregate growth. The relative price of services is falling, whereas the relative price of agriculture is rising, implying that productivity growth is lowest in services. Again, the high-skilled population share is closely related to the skill premium, which is rising over time. However, given that skill-biased technical and structural change cause a rise in the skill premium, the model requires the high-skilled population share to be rising over time, in line with the data as we discuss below. Lastly, the calibration for the government parameters is as in the base year 1980. Table 4.2 shows that the gap between the AMTR faced by the top

Table 4.3 Parameters for the US Calibration

Parameter	Interpretation	Value
Preferences		
σ	Elasticity of substitution	0.300
ε_A	Income elasticity agriculture	0.100
ε_G	Income elasticity goods	1.000
ε_S	Income elasticity services	1.800
Ω_A	Level parameter agriculture	0.008
Ω_G	Level parameter goods	1.000
Ω_S	Level parameter services	19.272
γ	Coefficient of risk aversion	2.000
φ	Labor supply elasticity	5.000
B	Labor disutility	4291.443
Production		
ρ	Elasticity of substitution	1.420
A_A	Neutral technology agriculture	0.157, 0.839, 3.411
A_G	Neutral technology goods	0.325, 0.836, 2.251
A_S	Neutral technology services	0.412, 1.000, 1.669
α_A	Skill intensity agriculture	0.126, 0.302, 0.351
α_G	Skill intensity goods	0.170, 0.299, 0.479
α_S	Skill intensity services	0.262, 0.440, 0.606
f_H	Population share high skilled	0.097, 0.238, 0.371
Government		
G_A	Government consumption agriculture	0.000, 0.001, 0.001
G_G	Government consumption goods	0.004, 0.006, 0.010
G_S	Government consumption services	0.007, 0.015, 0.026
T	Transfer	0.004, 0.011, 0.016
λ	Tax function level	0.210, 0.229, 0.203
τ	Tax function progressivity	0.260, 0.262, 0.163

Notes: This table summarizes the calibrated parameters for the U.S. For the time-varying parameters the values correspond to the calibration for the years 1950, 1980, and 2010.

10% compared to the bottom 90% is roughly constant between 1950 and 1980, while it is significantly lower in 2010. This translates into similar progressivity parameters in 1950 and 1980, but a significantly lower one in 2010. This is in line with the estimated time series for progressivity based on the loglinear tax function in Ferriere and Navarro (2020) also based on the data by Mertens and Montiel Olea (2018). Heathcote, Storesletten, and Violante (2020b), by contrast, estimate a relatively stable progressivity parameter for the loglinear tax function between 1980 and 2010. However, a key difference is that we model transfers separately. While progressivity of purely the income tax function decreases in our calibration, redistribution through transfers increases over time.

Table 4.4 Untargeted Moments in the US calibration

Moment	**Data**	**Model**
Service share 1950	59.79%	61.05%
Manufacturing share 1950	33.31%	33.11%
Service share 2010	81.04%	79.27%
Manufacturing share 2010	17.93%	19.84%
High-skilled population share 1950	8.56%	9.73%
High-skilled population share 1980	21.59%	23.88%
High-skilled population share 2010	34.98%	37.12%

Notes: This table compares untargeted moments with the model implied values for the U.S. calibration.

Table 4.4 shows the model fit for a number of untargeted moments. First, while we target the sector shares in 1980 in our calibration, we do not target the sector shares in 1950 and 2010. The model still matches these sector shares very well. It captures the 20 percentage point rise in the service share from 1950 to 2010 and the associated drops in the agriculture and goods shares. Capturing these structural changes well is key for our analysis of optimal taxes and transfers over time. The very good model fit for the sector shares gives us confidence that the non-homothetic CES preferences, parameterized according to the estimates of Comin, Lashkari, and Mestieri (2021), capture the degree of non-homotheticities very well.

Table 4.4 also shows the model implied high-skilled population shares over time. We need these as free parameters in the model calibration to match the skill premium exactly. The implied values are very close to the data. While there is a close relationship between the explicitly targeted high-skilled income shares by sector and the skill premium on the one hand and the high-skilled population share on the other hand, the fit does not have to be perfect because we impose the same skill premium across sectors in the model, which is not exactly the case in the data. Therefore, it is reassuring that the model implied population shares are very close to the data.

Finally, the model also captures the fact that labor supply is falling over time. Bick, Fuchs-Schündeln, and Lagakos (2018) show for a cross-section of countries that hours worked tend to be higher for countries at early stages of development compared to more developed countries. Boppart and Krusell (2020) document falling hours in the time series for the U.S. and a variety of other developed countries. We also see a fall in hours worked in the Census data.

4.4.6 Optimal Tax-and-Transfer System for the U.S.

The Ramsey problem in this economy is to choose the parameters of the tax-and-transfer system λ, τ, T to maximize welfare. As in the partial equilibrium setup, we follow an inverse optimum approach. To implement this, we define Pareto weights as

$$f(\theta w) = \mu + (\theta w)^{\nu}. \tag{4.21}$$

Pareto weights are a function of the product of the skill specific wage and the within skill efficiency units of labor. We set parameters μ and ν of the Pareto weight function such that the calibrated 1950 tax system is optimal at the observed skill premium. To make the 1950 tax-and-transfer system optimal the planner has to put a lower weight on low income and a higher weight on high income households than a utilitarian planner would do. For the years 1980 and 2010, we have to decide which Pareto weights to use. One option would be to apply the Pareto weight function using the parameters μ and ν required for 1950 to the new skill premium in 1980 and 2010. This has the disadvantage that given the estimated Pareto weights for 1950 we would increase the weights the planner puts on richer households, partially undoing the effect of the non-homotheticities on desired redistribution. Simply keeping the Pareto weights constant across groups even though the skill premium changed has the disadvantage that the planner may put different weights on individuals with different skill levels, who have the same income. Hence, as our benchmark we choose to keep μ constant across years but to adjust ν such that the ratio between the weight on the richest group and the weight on the poorest group remains the same.

In Figure 4.4 we show the optimal average tax rates implied by the entire tax-and-transfer system for income deciles for the three years.[8] The benchmark year is 1950, for which the optimum corresponds to the calibrated system. The overall tax-and-transfer system is progressive in the sense that average tax rates are increasing with income. In the lowest income decile the average tax rate is negative, implying that households receive a net transfer.

The optimal tax-and-transfer system in 1980 is less redistributive. The net transfer for the bottom income decile is essentially zero. Tax rates are slightly higher also for the other bottom income deciles. The top income decile, by contrast, faces a lower optimal tax rate. Which changes drive these results? The skill premium and the share of high-skilled individuals increase from 1950 to 1980. Standard optimal tax theory abstracting from level effects thus would imply that more redistribution is optimal. However, the non-homotheticity effect overcompensates the standard channel. Consumption of

[8] We replicate this figure in Appendix D.2.4 for the case in which we simply keep Pareto weights the same across groups in later years. There is no meaningful difference in optimal tax rates across the two cases.

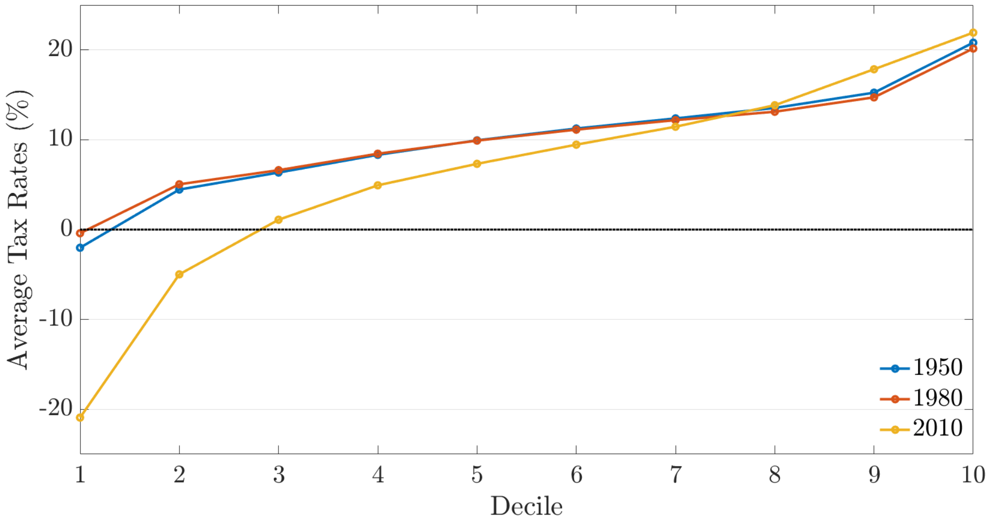


Figure 4.4 Optimal Average Tax Rates

Notes: This figure shows optimal average rates given by the entire tax-and-transfer system by income decile for the U.S. in 1950, 1980, and 2010.

necessities in form of agricultural goods drops between 1950 and 1980 and consumption of the relative luxury good services rises. Therefore, the planner has a lower desire for redistribution, explaining why the optimal tax-and-transfer system becomes less redistributive.

Until 2010, however, this change is more than reversed. The rise in inequality is much stronger over this later time period, which strongly affects the desire to redistribute. Moving from 1980 to 2010, the effect of non-homotheticities is also weaker relative to the change between 1950 and 1980 because the economy is already starting from a higher level.

In Figure 4.5 we perform a decomposition of the change between 1950 and 2010. Over the entire time period the tax-and-transfer system becomes more progressive in average rates. We proceed in five steps. The starting point is to find the optimal tax-and-transfer system in 1950 in partial equilibrium such that we can then shut off and on different channels in the next steps. Thanks to the inverse optimum approach and relatively weak general equilibrium effects of taxes on the skill premium (see discussion below), the optimal partial equilibrium system that we find as starting point for this exercise is virtually indistinguishable from the optimal general equilibrium system shown in Figure 4.4.

In the second step we reduce all prices by the same factor to capture the amount of aggregate growth that took place between 1950 and 2010. Households are richer now, but they still face the same relative prices. Also, relative inequality in incomes is unchanged. This isolates the non-homotheticity effect. This uniform price change has a sizeable effect on optimal redistribution. Because the economy is much

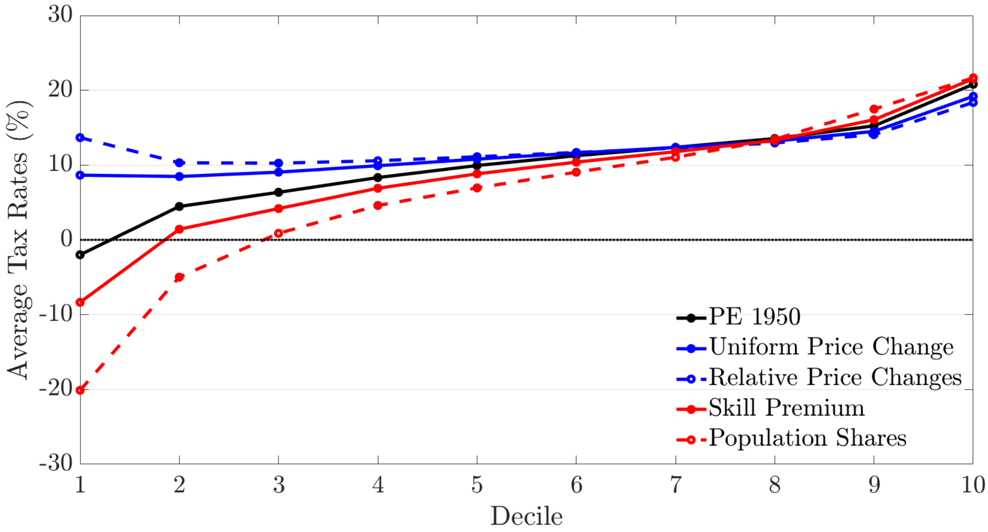


Figure 4.5 Optimal Average Tax Rates: Decomposition

Notes: This figure decomposes the change in optimal average tax rates between 1950 and 2010.

richer after this price change consumption shifts towards luxuries and the planner's desire to redistribute declines. Optimal average tax rates are only mildly progressive with much higher tax rates in the bottom income deciles.

The third step of the decomposition is to also change relative prices. Because agricultural goods get cheaper over time and services get more expensive this is another force for less desired redistribution. The goods that are disproportionately consumed by the poor are getting cheaper compared to the previous step, whereas the rich face higher prices of their consumption baskets. Hence, the planner does not have to implement as much redistribution.

Next, we adjust the skill premium to its 2010 level. This implies a higher level of inequality because the skill premium rises from 1.44 to 2.06. As expected, this increases the progressivity of the overall tax-and-transfer system. Tax rates for the bottom deciles fall, while tax rates at the top rise.

The effect of changing the skill premium is of similar size as the effect of adjusting population shares from the 1950 to the 2010 levels, which is the last step of the decomposition. This also significantly increases the optimal net transfers received by the poor and increases tax rates at the top. A larger share of high skilled individuals raises income inequality and the number of individuals from which high taxes can be raised, explaining the large changes to the tax system.

With this last step we are almost back at the 2010 optimal tax-and-transfer system, even though we are ignoring general equilibrium effects of taxes in this decomposition. This suggests that general equilibrium effects of tax changes are relatively weak in this model. This is the case because there is a

Table 4.5 Moments for the China Calibration

Moment	**1989**	**1999**	**2009**
Skill premium	1.00	1.10	1.58
High-skill inc. share agriculture	2.00%	4.00%	8.00%
High-skill inc. share manufacturing	2.00%	6.00%	14.00%
High-skill inc. share services	11.00%	18.00%	34.00%
Service share	32.89%	38.57%	44.41%
Manufacturing share	42.50%	45.36%	45.96%
Real GDP per capita	0.41	1.00	2.45
Rel. price agriculture to manufacturing	1.10	1.00	0.90
Rel. price services to manufacturing	0.90	1.00	1.10
G/Y	13.00%	13.00%	13.00%
T/Y	1.00%	1.00%	1.00%
AMTR Top 10%	14.00%	14.00%	14.00%
AMTR Bottom 90%	14.00%	14.00%	14.00%

Notes: This table summarizes the moments for the China calibration. Real GDP per capita and relative prices are normalized to 1 in 1999.

lot of overlap in the income distributions of low and high skilled individuals. Therefore, changing the tax systems does not induce large changes in the relative labor supply of low skilled versus high skilled, so that the skill premium moves little. It requires much larger changes to the tax system to observe significant changes to the skill premium.

4.4.7 Calibration for China

We now turn to an alternative calibration of the model to the Chinese economy. This is of interest because we have seen that the role of non-homotheticities is larger at earlier stages of development. The current calibration is relatively rough because some of the required data is less easily available for China than for the United States. However, the calibration captures key features of structural change and the dynamics of inequality in China. Also because of limited data availability we restrict ourselves to the years 1989, 1999, 2009.

Table 4.5 shows the data moments we are targeting for China. For the aggregate growth part of the data moments, we use data from the World Development Indicators for the sectoral contributions to total value added. The service sector expanded from 33% in 1989 to 44% in 2009. Over the same time period also the goods sector grew slightly from 43% to 46%. Correspondingly, the agricultural sector shrank from 25% to 10%. This shows the potential for an even larger effect of non-homotheticities, as households spend a large share of their incomes on necessities initially.

Relatedly, even though we are considering a shorter time period than for the U.S., aggregate growth was much larger for China. We take data for the growth rate of real GDP per capita from Fang and Herrendorf (2021). GDP per capita rose by a factor of six. We also use data from this paper to infer targets for the high-skilled income shares by sector, which determines the skill intensities in the model. Because their sector definitions are different from ours, we cannot simply take their data. Hence, the targets reported in Table 4.5 are approximations, but should be roughly in line with the true values. As for the U.S., services are the high-skill intensive sector and skill intensities are going up over time. We also use the evidence from Fang and Herrendorf (2021) to discipline the skill premium. In 1989, there was no skill premium, but it increased to 1.58 in 2009, with the larger part of this increase happening in the second half of the sample.

Finally, when calibrating the government parameters we have to take a stand on the size of the welfare state, total government spending, and the progressivity of taxes. While China has a fairly progressive income tax that has become more important over time (Li and Ma, 2017; Piketty and Qian, 2009), it raises the majority of its tax revenue through corporate taxes and value added taxes. As value added taxes are generally considered to be less progressive than income tax, we decide to target a flat tax rate in all periods. The total amount of revenue raised is roughly constant over time relative to GDP, so we also keep this constant at 14% over time. The Chinese welfare state has expanded, but a large share of this goes to the pension system, which we do not model. Hence, we calibrate a constant split between government consumption and transfers, with G/Y of 13% and T/Y of 1%.

Our calibration strategy to match these targets closely follows what we have done for the U.S. However, we choose slightly different parameters for the non-homothetic CES utility function, within the range of estimates by Comin, Lashkari, and Mestieri (2021). We could alternatively keep them at their U.S. levels, but adjusting them slightly helps improving the match for all the sector shares. Also, we choose the evolution of sectoral relative prices such that we match the sector shares in all periods well. As for the U.S., we match the middle year sector shares exactly, but here we also target the sector shares in the other years explicitly.

4.4.8 Optimal Tax-and-Transfer System for China

The optimal tax-and-transfer system for China is shown in Figure 4.6. As for the U.S., we follow an inverse optimum approach. We pick as our base year 1999, so the optimal tax-and-transfer system for the year 1999 corresponds to our calibration. It is mildly progressive in overall average tax rates of the entire tax-and-transfer system because there is a small lump sum transfer combined with a flat tax.

Table 4.6 Parameters for the China Calibration

Parameter	Interpretation	Value
Preferences		
σ	Elasticity of substitution	0.500
ε_A	Income elasticity agriculture	0.100
ε_G	Income elasticity goods	1.000
ε_S	Income elasticity services	1.300
Ω_A	Level parameter agriculture	0.081
Ω_G	Level parameter goods	1.000
Ω_S	Level parameter services	1.365
γ	Coefficient of risk aversion	2.000
φ	Labor supply elasticity	5.000
B	Labor disutility	2504.842
Production		
ρ	Elasticity of substitution	1.420
A_A	Neutral technology agriculture	0.227, 0.744, 2.514
A_G	Neutral technology goods	0.250, 0.794, 2.614
A_S	Neutral technology services	0.359, 1.000, 3.092
α_A	Skill intensity agriculture	0.061, 0.099, 0.170
α_G	Skill intensity goods	0.061, 0.129, 0.242
α_S	Skill intensity services	0.187, 0.261, 0.418
f_H	Population share high skilled	0.048, 0.095, 0.160
Government		
G_A	Government consumption agriculture	0.002, 0.004, 0.007
G_G	Government consumption goods	0.042, 0.011, 0.027
G_S	Government consumption services	0.033, 0.009, 0.025
T	Transfer	0.004, 0.003, 0.003
λ	Tax function level	0.140, 0.140, 0.140
τ	Tax function progressivity	0.000, 0.000, 0.000

Notes: This table summarizes the calibrated parameters for China. For the time-varying parameters the values correspond to the calibration for the years 1989, 1999, and 2009.

Moving back in time to 1989 is associated with a slightly lower skill premium. Also, to match the skill premium at calibrated skill intensities our model requires the share of highly skilled individuals to go up over time (in line with the data). For these reasons, standard tax theory would predict that less redistribution is desirable in 1989. However, there is also a large increase in average income and a significant shift away from the consumption of necessities from 1989 to 1999. This latter effect dominates. Instead of paying net taxes as in the 1999 (inverse) optimum, the lowest income decile would receive a net transfer. Average tax rates are also lower for the next income deciles and higher at the top.

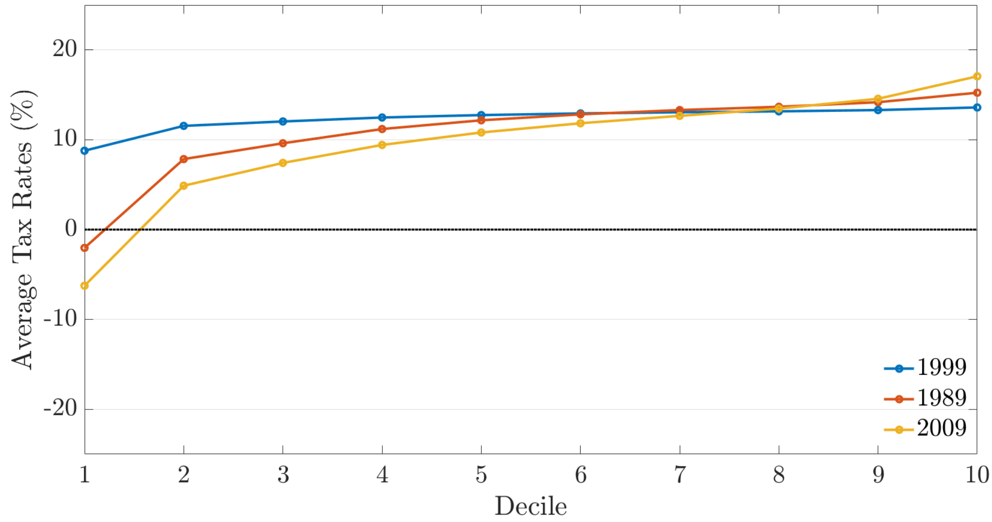


Figure 4.6 Optimal Average Tax Rates

Notes: This figure shows optimal average rates given by the entire tax-and-transfer system by income decile for China in 1989, 1999, and 2009.

The optimal tax-and-transfer-system is also more redistributive in 2009 compared to 1999. The economy is again much richer on average and particularly the bottom income groups have moved further away from absolute poverty. They spend a smaller share of their income on agricultural goods. However, there is also a larger share of high-skilled who earn a much higher skill premium. Thus, the planner prefers to impose a higher tax rate at the top. This allows to give a larger transfer providing more redistribution towards the bottom of the distribution. When comparing the optimal 1989 and 2009 systems, the 2009 system is only slightly more redistributive even though inequality is much larger. The effect of individuals growing out of absolute poverty with aggregate growth and non-homothetic preferences almost cancels out the rise in inequality.

As our benchmark, we again adjusted Pareto weights for the other years as we did in the U.S. case, keeping the ratio of the Pareto weight on the highest earning group to that of the lowest earning group constant. In Figure D.2 in the appendix we repeat the exercise for Pareto weights that are simply kept constant for groups characterized by a skill level and within-skill productivity. This makes a larger difference for the results than in the U.S. case. However, the main message is unaffected: Aggregate growth in combination with non-homothetic preferences lowers the benefits of redistribution. Quantitatively, the effect is even stronger, with the welfare state optimally becoming less generous from 1989 to 2009.

4.5 Conclusion

In this paper we incorporate the optimal income taxation problem into a state-of-the-art model of structural change. This allows us to show that aggregate growth matters for optimal redistribution, beyond standard effects through relative inequality. With non-homothetic preferences, the taste for redistribution declines with aggregate growth as households move away from absolute poverty and shift their consumption away from necessities towards luxuries. However, aggregate growth and structural change have additional effects on the optimal design of the welfare state through changing prices of different commodities and changing inequality.

In our calibrated economies, growth has non-monotonic effects on the optimal size of the welfare state. At early stages of development the effect of non-homotheticicities is dominant, as many people have to spend a large share of their incomes on necessities. At later stages of development this effect weakens and relative inequality concerns become dominant.

These patterns naturally raise the question whether using public debt to finance the welfare state at early stages of development could be beneficial. We explore this possibility in ongoing work.

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Appendix A

Appendix to Chapter 1

A.1 Data

For the empirical investigation we combine data from several Danish registry data sets provided by Statistics Denmark.

BFL. The main data source on earnings and employment in this paper is the BFL (*Beskæftigelse for Lønmodtagere*). This data set contains information on employment spells within each month from 2008 until 2018. For each of these spells we observe earnings (including and excluding pension contributions), hours (reported hours and an imputed measure), and the start and end date of the spell. Also, for each individual there is an identifier based on the anonymized social security number, which can be used to link this individual to other data sets. Similarly, for each firm there is an identifier that also can be used to match the data to other data sets. It also contains information on occupation, industry, and location.

BEF. The BEF (*Befolkningen*) data set contains information on demographics, which can be merged to with the BFL data. A control variable we use from this data source is an individual's sex.

IDAP. Another individual level data set is IDAP (*Integreret Database for Arbejdsmarkedsforskning - Persondata*) from the integrated database for labor market research. From this data set we take variables on individuals' age and labor market experience.

UDDA. The UDDA (*Uddannelser*) database contains information on individuals' education.

IND. The IND (*Indkomst*) database contains information on pre- and after-tax income which we use to estimate the parameters of the tax function.

FIRM. The FIRM (*Generel firmastatistik*) database contains general information on firms. From this database we take information on firm-level employment and information on the source of firm-level accounting data in the FIRE database (see below).

FIRE. FIRE (*Regnskabsstatistikken*) is the database for firm-level accounting information. From this source we take information on firm level revenues, value added, and profit.

FIKS. A complementary source for firm revenue information is the FIKS (*Firmaernes køb og salg*) database. This data set includes information on firms' purchases and sales from value added tax data. In addition to the purchase and sales numbers it contains information on the frequency at which a firm settles its VAT accounts. For the largest firms this is done at monthly frequency; for smaller firms it is done at quarterly or half-yearly frequency.

A.2 Additional Evidence

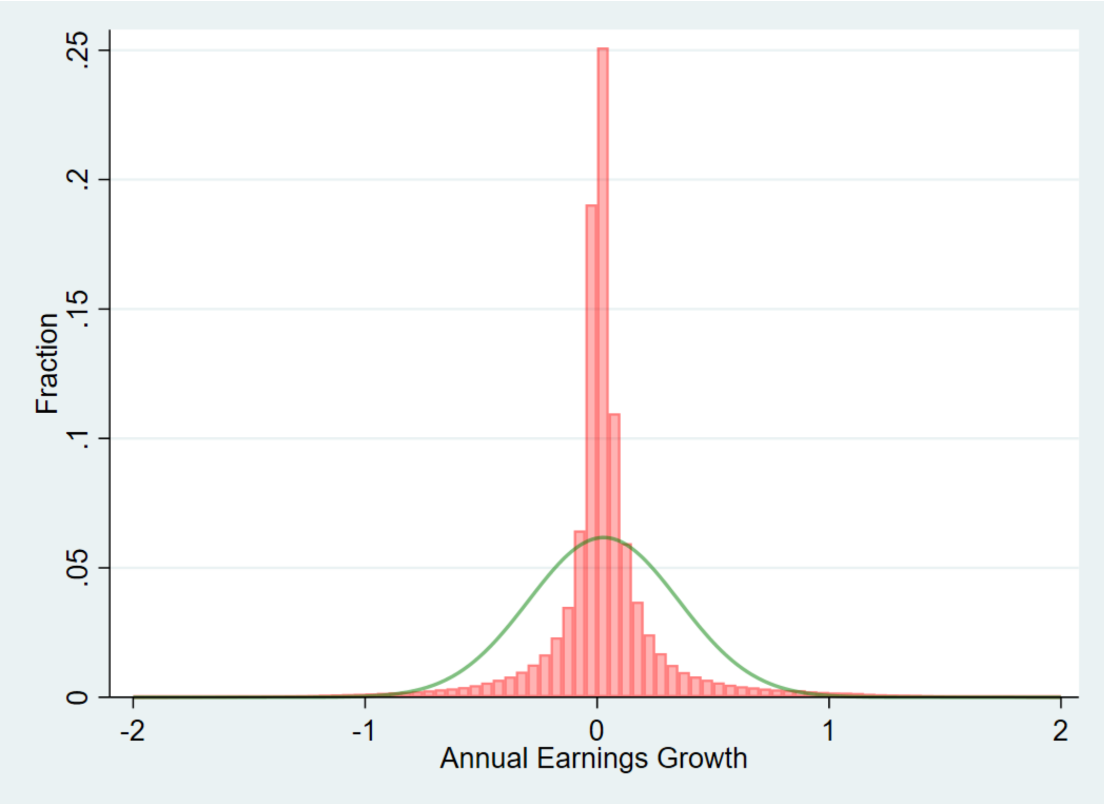


Figure A.1 Annual Earnings Growth Distribution

Notes: This figure shows the annual earnings growth distribution for the entire sample. Growth rates are based on raw labor earnings. The green line compares a normal distribution with the same mean and standard deviation.

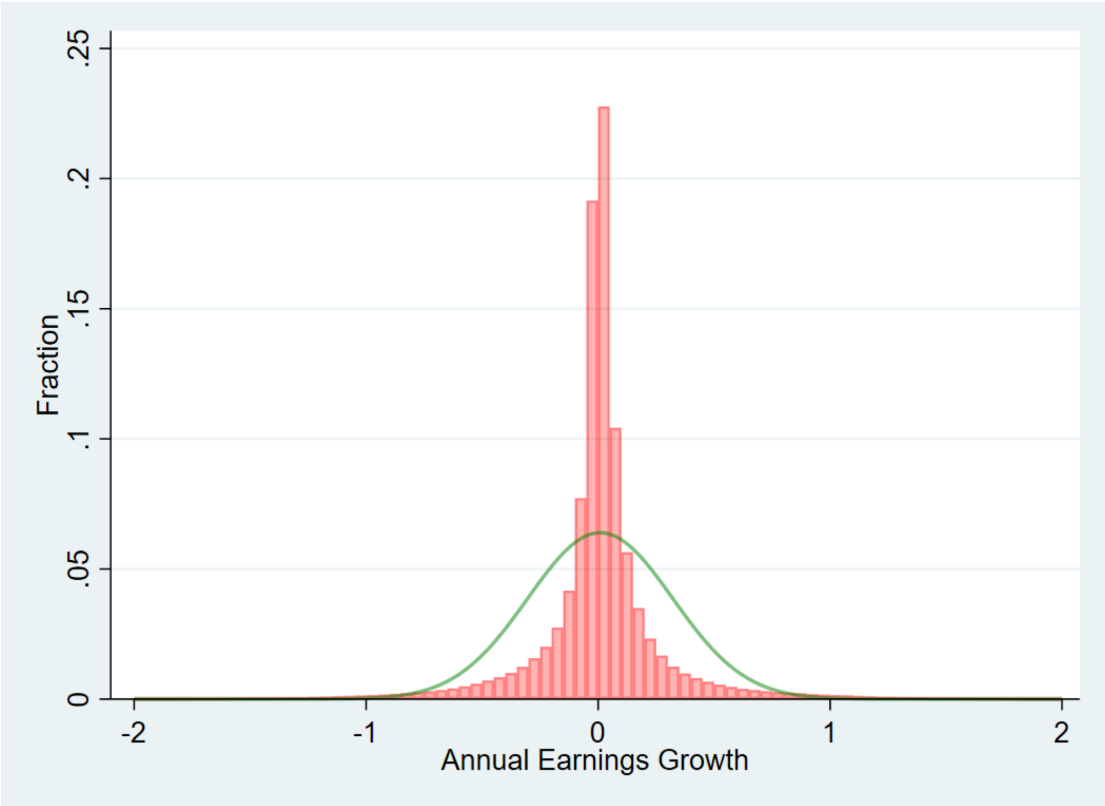


Figure A.2 Annual Earnings Growth Distribution

Notes: This figure shows the annual earnings growth distribution for the entire sample. Growth rates are based on residual earnings taking out age, gender, education, occupation, industry, and location effects. The green line compares a normal distribution with the same mean and standard deviation.

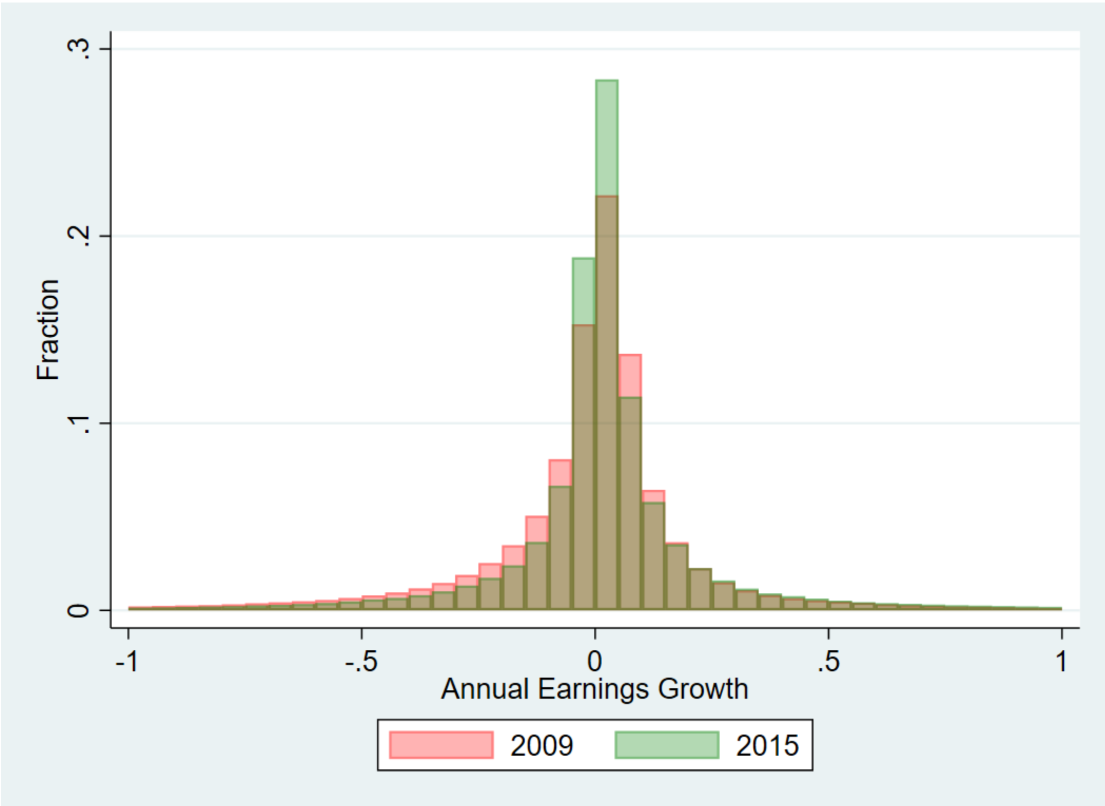


Figure A.3 Annual Earnings Growth Distribution: Business Cycle

Notes: This figure shows the annual earnings growth distribution for the years 2009 (recession) and 2015 (expansion). Growth rates are based on residual earnings taking out age, gender, and education effects.

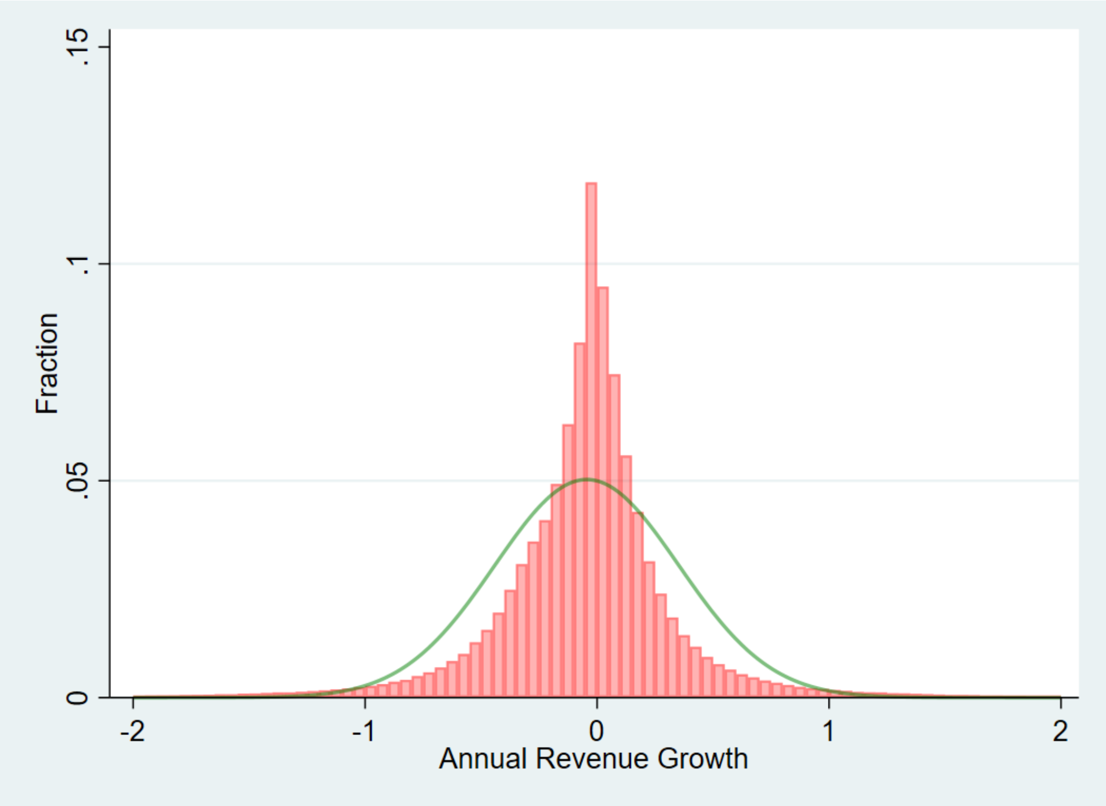


Figure A.4 Annual Revenue Growth Distribution

Notes: This figure shows the annual revenue growth distribution. This is based on data from value added tax data, which is available for more firms than the accounting information used in the main text. The green line compares a normal distribution with the same mean and standard deviation.

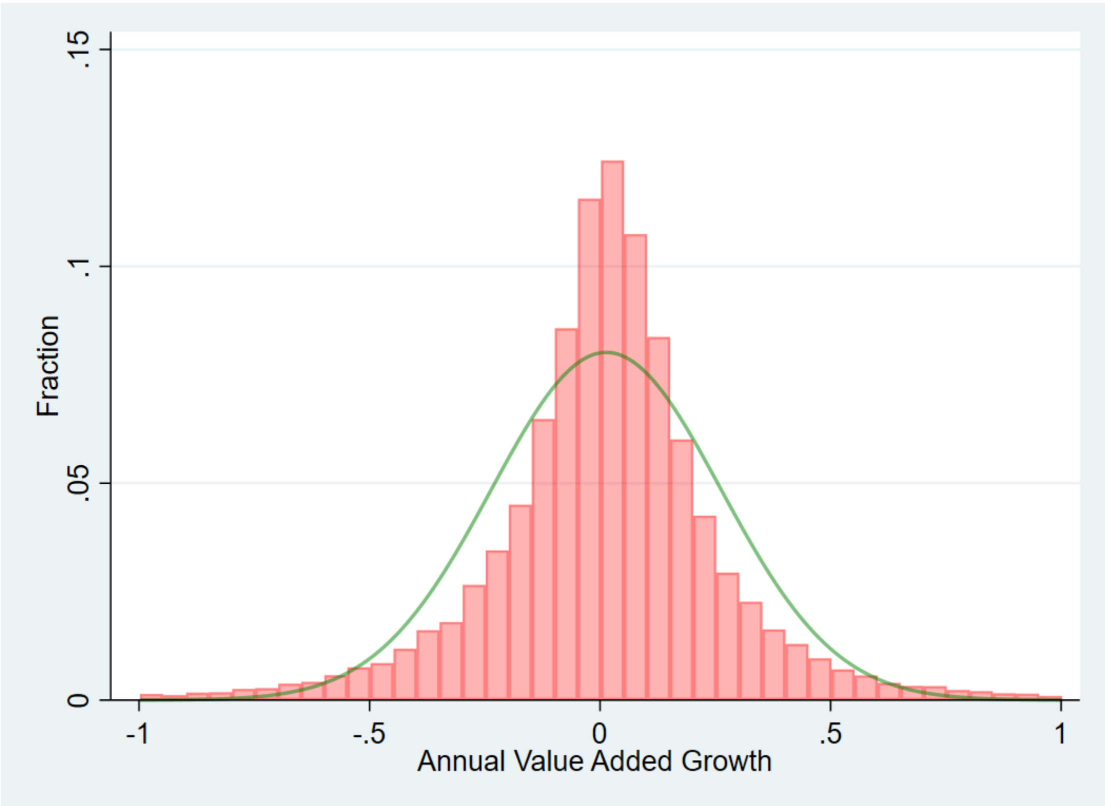


Figure A.5 Annual Value Added Growth Distribution

Notes: This figure shows the annual value added growth distribution. The green line compares a normal distribution with the same mean and standard deviation.

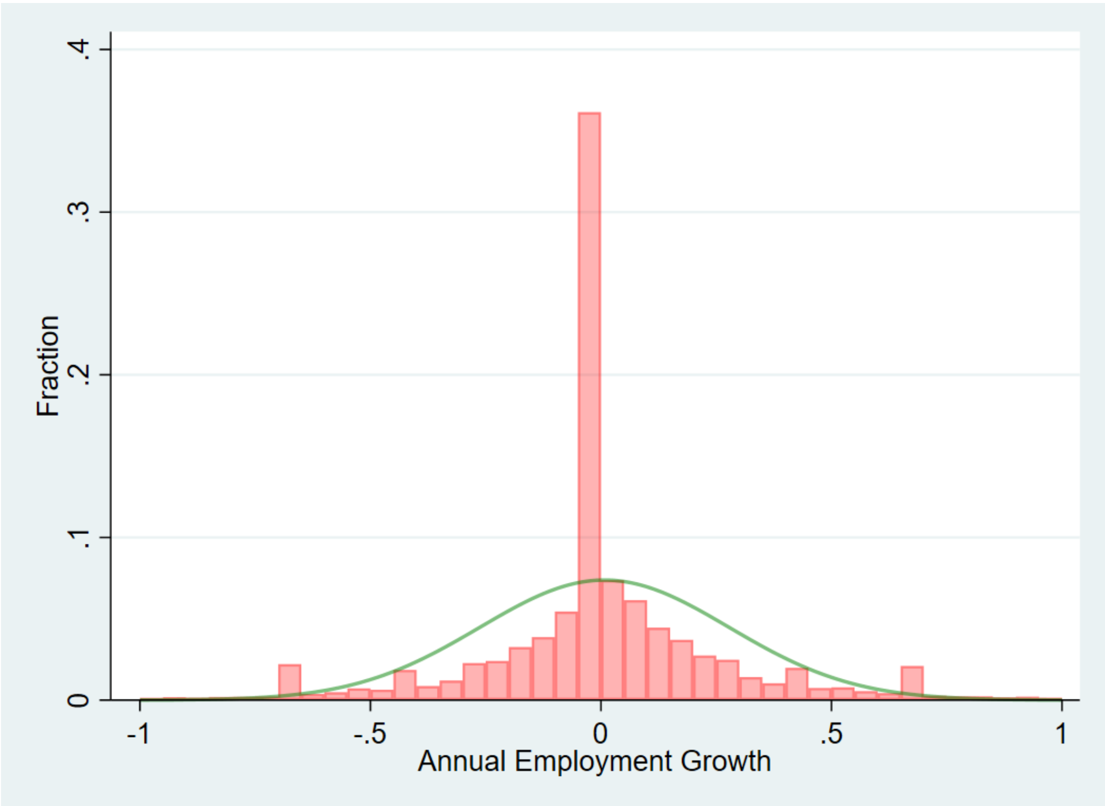


Figure A.6 Annual Employment Growth Distribution

Notes: This figure shows the employment growth distribution. The green line compares a normal distribution with the same mean and standard deviation.

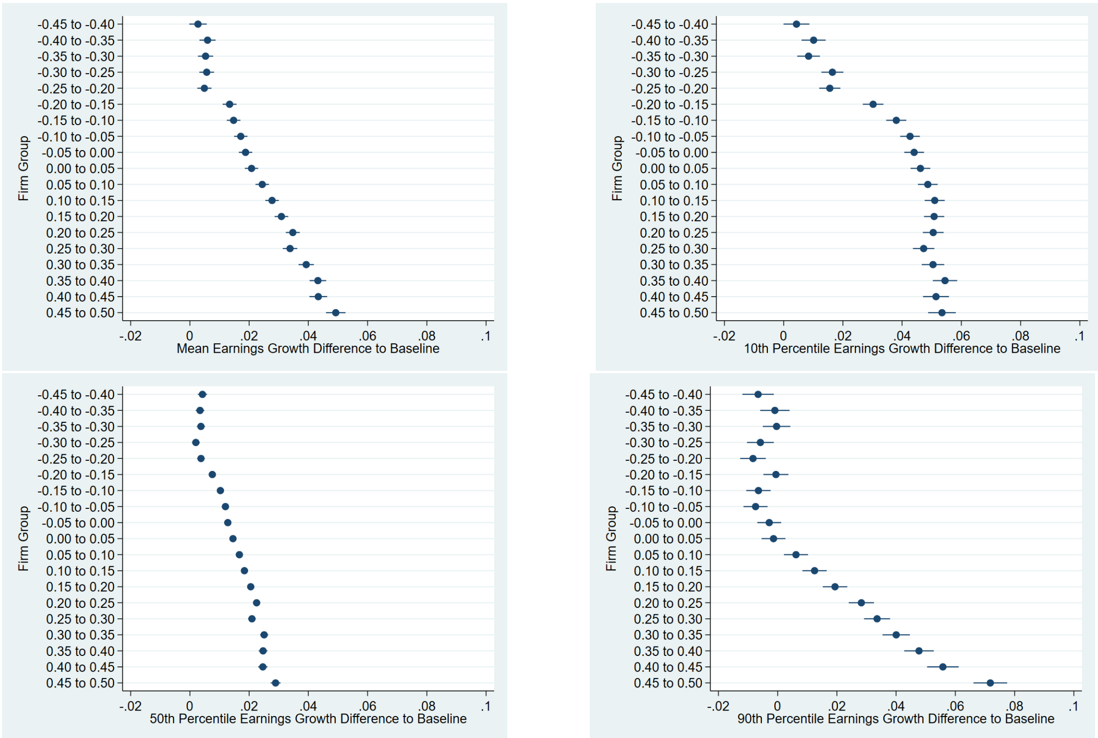


Figure A.7 Worker Earnings Growth by Firm Revenue Growth: Stayers

Notes: This figure shows the the relationship between firm performance and different parts of the earnings growth distribution for individuals who are continuously employed at the same firm for two consecutive years.

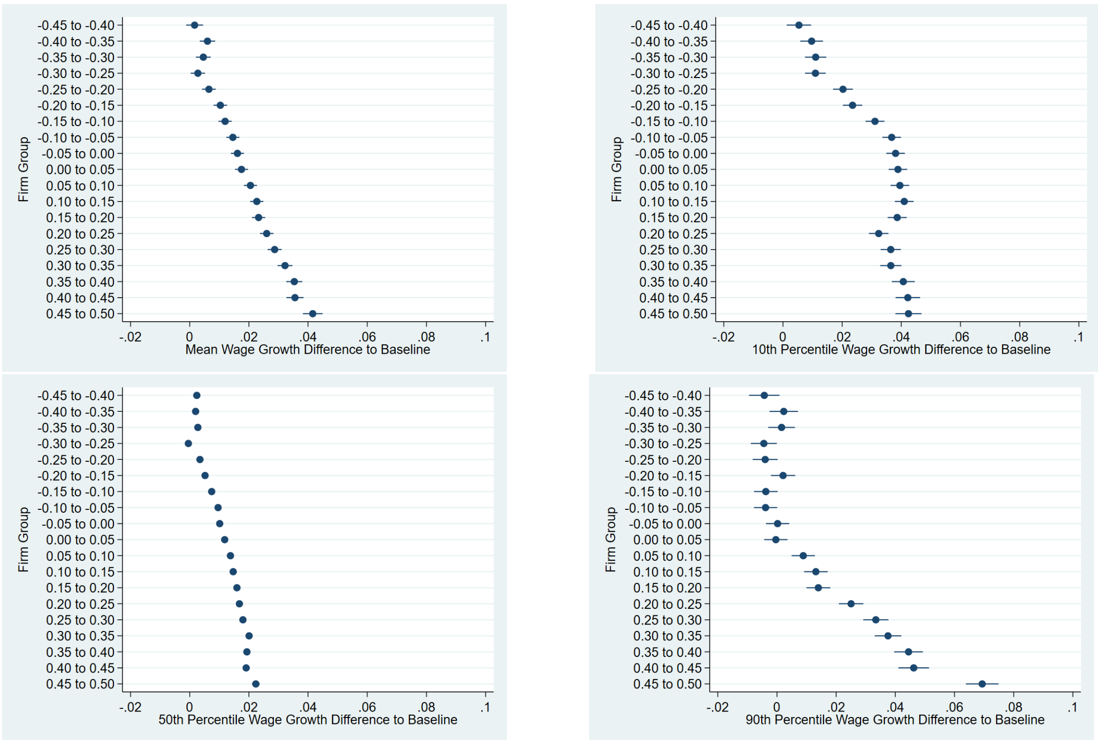


Figure A.8 Worker Wage Growth by Firm Revenue Growth: Stayers

Notes: This figure shows the the relationship between firm performance and different parts of the wage growth distribution for individuals who are continuously employed at the same firm for two consecutive years.

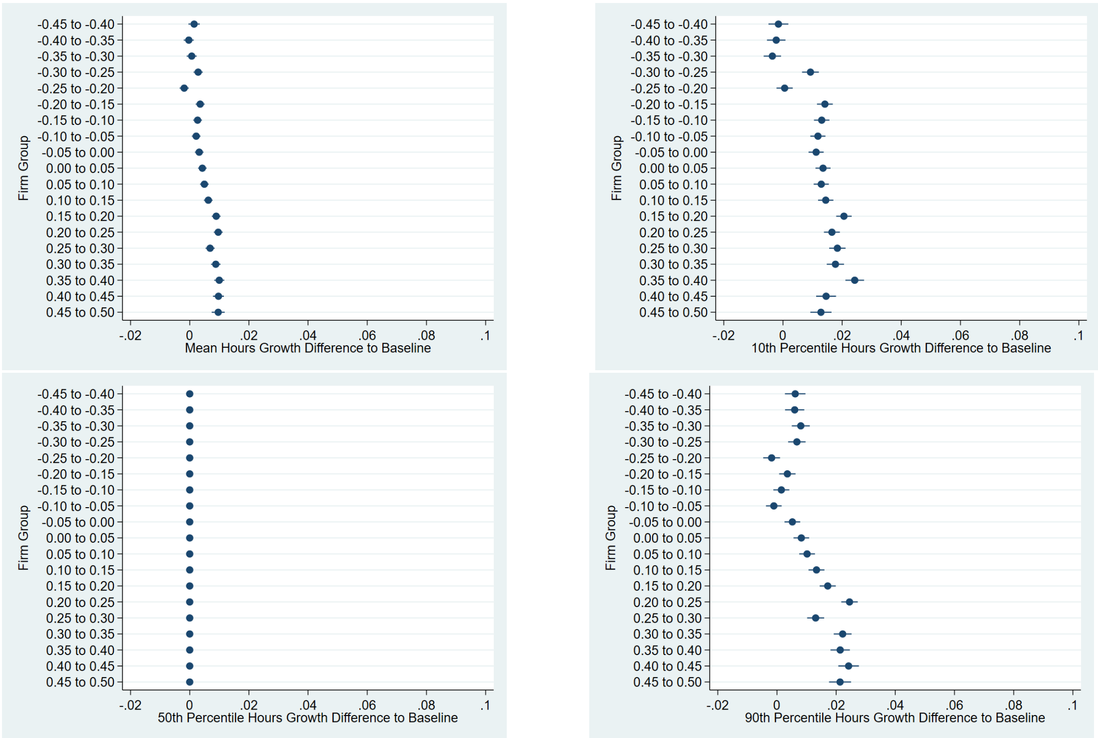


Figure A.9 Worker Hours Growth by Firm Revenue Growth: Stayers

Notes: This figure shows the the relationship between firm performance and different parts of the hours growth distribution for individuals who are continuously employed at the same firm for two consecutive years.

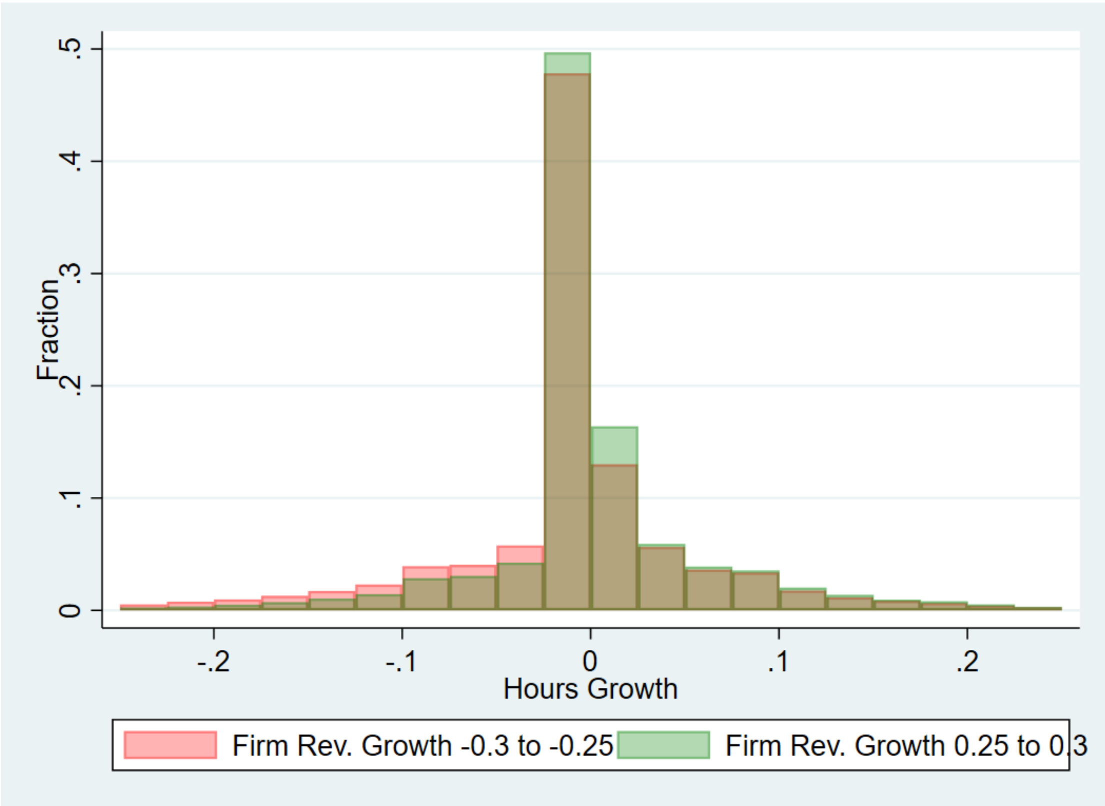


Figure A.10 Worker Hours Growth by Firm Revenue Growth: Stayers

Notes: This figure shows the hours growth distribution conditional on firm revenue growth for workers who are continuously employed at the same firm for two years. Hours are based on an hours measure imputed by Statistics Denmark.

A.3 Additional Evidence from Germany

In this appendix we provide supporting evidence from German matched employer-employee data showing that key features we describe using Danish data are not unique to the Danish institutional environment. We use Denmark as our benchmark as the data has some advantages, which will become clear below. Still, it is encouraging that key patterns also hold in the German context.

A.3.1 Data

Data Set. We use data provided by the Institute for Employment Research at the German Federal Employment Agency. Specifically, we use the "Linked Employer-Employee Data from the IAB" (LIAB) Longitudinal Model 1993-2014.[1] This data set combines establishment level information from the IAB Establishment Panel with information on employees drawn from social security records.

The IAB Establishment Panel is a representative survey of German establishments[2] with at least one employee subject to social security.[3] It is a yearly panel survey. The first survey was conducted in 1993 with roughly 5000 establishments. The number of interviewed establishments rose to around 15000 until 2000 and stayed constant from then on. To ensure high data quality, the vast majority of the interviews is conducted face-to-face at the establishments. The main variable of interest for the analysis here is an establishment's business volume.

The employee data of the LIAB is drawn from social security records. For the LIAB Longitudinal model a subsample of establishments from the IAB establishment panel is chosen. Then, every worker who is employed at a selected establishment for at least one day in the relevant time period is included in the worker sample. For every individual included in the sample every employment and unemployment spell for the period from 1993 to 2014 is added independently of whether the corresponding spell is at one of the covered establishments. Therefore, it is possible to observe wages and benefits for individuals before they join and after they leave a sample establishment. This is crucial to compute earnings growth at the individual level. The wage and benefit data is of high quality as it is drawn from social security records. This data is very accurate as misreporting by employers is penalized. The data set contains data on roughly 1-1.5 million individuals per year.

[1]For a detailed description of the LIAB Longitudinal Model, see Heining, Klosterhuber, and Seth (2014) and Heining, Klosterhuber, Lehnert, and Seth (2016).

[2]The data contains information on establishments and it is not possible to link establishments belonging to the same firm. We use the terms firm and establishment interchangeably, even though they are clearly distinct concepts.

[3]For details on the IAB Establishment Panel, see Ellguth, Kohaut, and Möller (2014).

Data Preparation and Sample Selection. Despite its very good accuracy, the use of German social security data has one drawback for the purpose of this project. Wages are censored at the social security contribution assessment ceiling. Only wages up to this threshold are subject to social security payments and if wages are higher, the social security limit is reported to the authorities. Therefore, we impute top wages. This is done by estimating a Pareto tail for the wage distribution and then drawing from the estimated Pareto distribution for those observations that are censored. This imputation is done separately by year, gender, and age groups. Also, it is done separately for East and West Germany, as the social security contribution limits differ. In order to not introduce excessive earnings changes through the imputation we follow Daly, Hryshko, and Manovskii (2018) and Busch, Domeij, Guvenen, and Madera (2018) in using a "fixed-effects" imputation: Only one random number is drawn per individual, which is then applied to the estimated Pareto distribution in any year this individual has a censored observation. We adjust worker earnings and firm revenues for inflation to obtain real earnings and revenues. Specifically, we adjust both using the GDP deflator to use a consistent measure.

We impose only some mild sample restrictions, in line with the literature. We restrict the sample to individuals older than 25 but younger than 60. This is done in order to avoid issues with individuals still attending college or retiring early. We also drop all marginal employment spells as these are only recorded by the social security administration since 1999. In addition, we drop all spells with a reported wage below the highest marginal employment threshold in the sample.

As we observe firm business volume at a yearly frequency, we also adopt yearly earnings as income concept. Therefore, we add up earnings from all spells to obtain yearly earnings. Following Busch, Domeij, Guvenen, and Madera (2018) we drop all individuals with annual earnings below 50% of 13 weeks of full-time work (520 hours) at the minimum wage.[4]

A.3.2 Evidence

In this section we include evidence on the worker earnings growth distribution depending on firm revenue growth. The analysis is performed as follows, as with the Danish data. Workers are matched to the firms they are employed with in period $t-1$. Establishments are grouped by their revenue growth from period $t-1$ to period t. There are ten groups, where five groups have negative revenue growth rates and five groups have positive growth rates. The first revenue growth group contains establishments with a drop in business of more than 20%. The next groups contain establishments with growth rates

[4]This sample selection procedure is similar but not exactly identical to the Danish case. This will be made consistent in future versions.

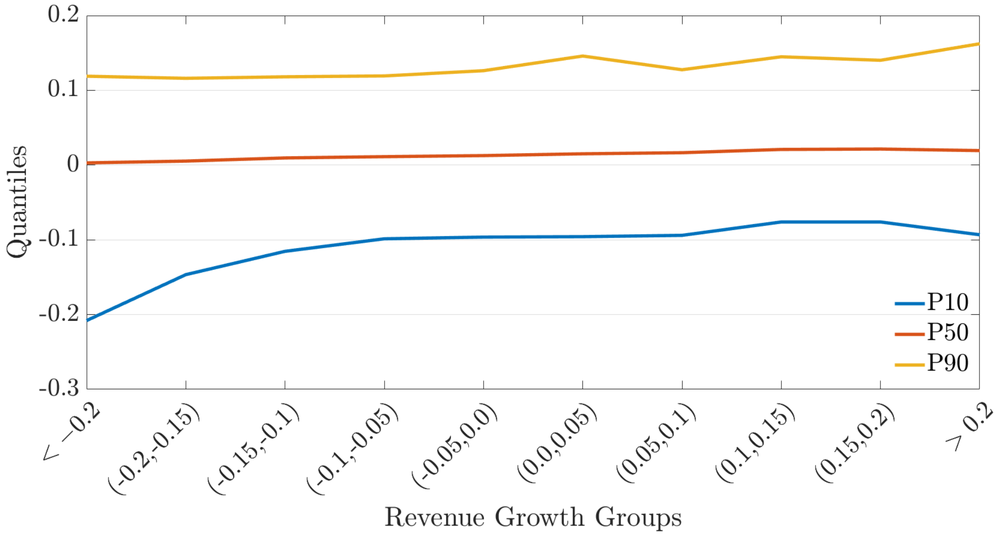


Figure A.11 Earnings Growth Distribution by Firm Growth for Germany

Notes: This figure shows percentiles of the annual earnings growth distribution of workers conditional on firm growth. The data is from the matched employer-employee data of the IAB.

between -20% and -15%, -15% and -10%, and so on. Then, moments of the worker earnings growth (also from $t-1$ to t) distribution are computed for each firm revenue growth group separately.

Figure A.11 shows the evidence for the sample where we do not condition on employment status in the second year. This corresponds to the information conveyed in Figures 1.5 and 1.6 with the Danish data.[5] As in the Danish context, there is a longer left tail to the earnings growth distribution in shrinking firms. The 10th percentile of the distribution is much lower in poorly performing firms. However, in all firms there are many workers whose earnings change very little, reflected in a median close to zero in all firm groups. In the fast growing firms, the 90th percentile of the earnings growth distribution is slightly higher.

Figure A.12 shows the same figure for individuals who are continuously employed at the same firm. As in the Danish case (Figure 1.7), the 10th percentile of the worker earnings growth distribution is higher across all firm groups because the largest earnings losses are associated with unemployment spells. However, it remains the case for stayers that in the shrinking firms there are more individuals with large earnings losses, reflected in a low 10th percentile of the earnings growth distribution, compared to growing firms.

[5]The presentation of the results is different across the two countries and will be made consistent when we next take out results again from the secure servers, which is, however, a time consuming process.

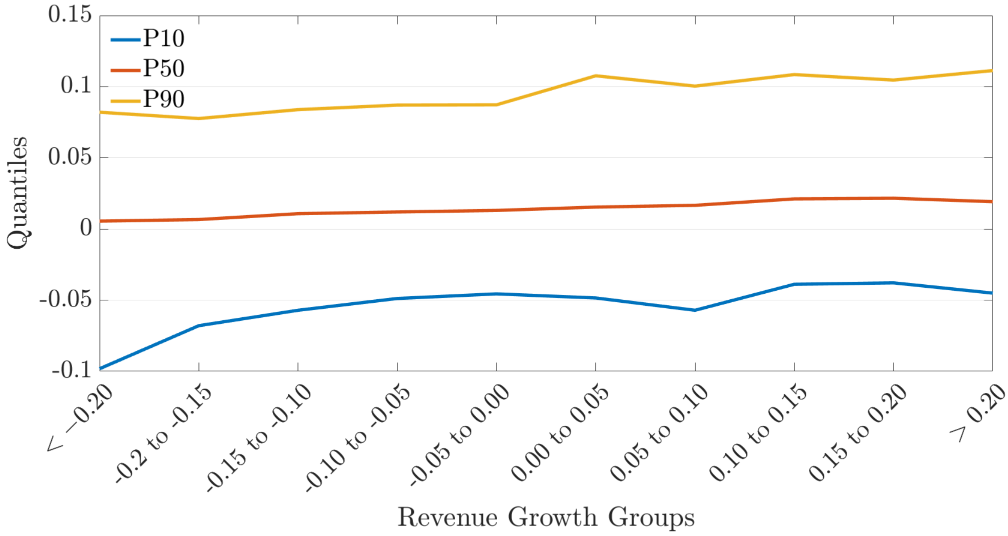


Figure A.12 Earnings Growth Distribution by Firm Growth for Germany: Stayers

Notes: This figure shows percentiles of the annual earnings growth distribution of workers who are continuously employed at the same firm conditional on firm growth. The data is from the matched employer-employee data of the IAB.

Appendix B

Appendix to Chapter 2

B.1 Empirical Robustness Exercises

B.1.1 Employed and Unemployed Spouses

Table B.1 Joint Labor Market Transitions (Full Sample): Spouse Unemployed

	Primary earner transition		
	EE	EU	EN
Cond. prob. of spousal UE transition	25.29%	26.27%	34.11%
Cond. prob. of spousal UU transition	61.97%	63.33%	46.01%
Cond. prob. of spousal UN transition	12.74%	10.41%	19.87%

Notes: This table shows the probability of a spousal transition from unemployment conditional on primary earner transitions for the entire population.

Table B.2 Joint Labor Market Transitions (Full Sample): Spouse Employed

	Primary earner transition		
	EE	EU	EN
Cond. prob. of spousal EE transition	97.61%	91.49%	88.84%
Cond. prob. of spousal EU transition	0.77%	5.78%	1.25%
Cond. prob. of spousal EN transition	1.62%	2.72%	9.92%

Notes: This table shows the probability of a spousal transition from employment conditional on primary earner transitions for the entire population.

B.1.2 Education

Table B.3 Joint Labor Market Transitions by Spousal Education

	Primary earner transition		
	EE	EU	EN
I. Spouse College Degree (All):			
Cond. prob. of spousal NE transition	6.91%	11.40%	20.88%
Cond. prob. of spousal NU transition	1.59%	6.43%	1.04%
Cond. prob. of spousal NN transition	91.50%	82.18%	78.08%
II. Spouse No College Degree (All):			
Cond. prob. of spousal NE transition	5.55%	7.20%	15.08%
Cond. prob. of spousal NU transition	1.65%	5.34%	1.45%
Cond. prob. of spousal NN transition	92.81%	87.46%	83.47%
III. Spouse College Degree (Young):			
Cond. prob. of spousal NE transition	7.31%	13.25%	33.25%
Cond. prob. of spousal NU transition	1.70%	7.22%	1.29%
Cond. prob. of spousal NN transition	90.99%	79.53%	65.46%
IV. Spouse College Degree (Old):			
Cond. prob. of spousal NE transition	6.04%	7.72%	11.81%
Cond. prob. of spousal NU transition	1.35%	4.87%	0.86%
Cond. prob. of spousal NN transition	92.61%	87.41%	87.33%
V. Spouse No College Degree (Young):			
Cond. prob. of spousal NE transition	6.30%	8.34%	21.76%
Cond. prob. of spousal NU transition	2.01%	6.28%	2.21%
Cond. prob. of spousal NN transition	91.69%	85.37%	76.03%
VI. Spouse No College Degree (Old):			
Cond. prob. of spousal NE transition	4.19%	4.20%	9.41%
Cond. prob. of spousal NU transition	0.99%	2.83%	0.80%
Cond. prob. of spousal NN transition	94.82%	92.97%	89.79%

Notes: This table shows the probability of a spousal transition from out of the labor force conditional on primary earner transitions by education of the spouse.

B.1.3 Cohort Effects

Table B.4 Joint Labor Market Transitions

	Primary earner transition		
	EE	EU	EN
I. Spouse is a Man (Young) :			
Cond. prob. of spousal NE transition	13.54%	14.07%	44.10%
Cond. prob. of spousal NU transition	6.19%	11.69%	2.59%
Cond. prob. of spousal NN transition	80.27%	74.24%	53.31%
II. Spouse is a Man (Old):			
Cond. prob. of spousal NE transition	4.50%	4.59%	10.36%
Cond. prob. of spousal NU transition	1.13%	3.23%	0.63%
Cond. prob. of spousal NN transition	94.37%	92.18 %	89.01%
III. Spouse born between 1960-70 (Young):			
Cond. prob. of spousal NE transition	6.98%	8.62%	21.67%
Cond. prob. of spousal NU transition	1.89%	6.70%	2.42%
Cond. prob. of spousal NN transition	91.13%	84.68%	75.92%
IV. Spouse born between 1960-70 (Old)			
Cond. prob. of spousal NE transition	4.28%	2.94%	12.86%
Cond. prob. of spousal NU transition	1.11%	3.68%	1.04%
Cond. prob. of spousal NN transition	94.61%	93.38%	86.10%

Notes: This table shows the probability of a spousal transition from out of the labor force conditional on primary earner transitions by gender and cohort.

B.1.4 Children

Table B.5 Joint Labor Market Transitions (< Age 40)

	Primary earner transition		
	EE	EU	EN
I. Have Children:			
Cond. prob. of spousal NE transition	6.26%	8.71%	28.30%
Cond. prob. of spousal NU transition	1.75%	6.65%	2.31%
Cond. prob. of spousal NN transition	91.98%	84.64%	69.40%
II. No Children:			
Cond. prob. of spousal NE transition	9.68%	12.68%	23.69%
Cond. prob. of spousal NU transition	3.40%	8.54%	1.59%
Cond. prob. of spousal NN transition	86.91%	78.78%	74.72%
III. Have Children below 5:			
Cond. prob. of spousal NE transition	5.63%	8.55%	30.09%
Cond. prob. of spousal NU transition	1.47%	6.14%	1.96%
Cond. prob. of spousal NN transition	92.90%	85.31%	67.95%
IV. No Children below 5:			
Cond. prob. of spousal NE transition	8.08%	9.95%	24.82%
Cond. prob. of spousal NU transition	2.60%	7.80%	2.35%
Cond. prob. of spousal NN transition	89.32%	82.24%	72.82%

Notes: This table shows the probability of a spousal transition from out of the labor force conditional on primary earner transitions by presence of children in the household.

B.1.5 Reasons for Non-Participation

Table B.6 Joint Labor Market Transitions

	Primary earner transition		
	EE	EU	EN
I. Excluding Retirement (Young):			
Cond. prob. of spousal NE transition	6.66%	9.32%	27.13%
Cond. prob. of spousal NU transition	2.00%	6.91%	2.06%
Cond. prob. of spousal NN transition	91.33%	83.77%	70.81%
II. Excluding Retirement (Old):			
Cond. prob. of spousal NE transition	4.95%	4.15%	11.45%
Cond. prob. of spousal NU transition	1.18%	3.33%	1.00%
Cond. prob. of spousal NN transition	93.87%	92.52%	87.54%
III. Excluding Disabled/Ill (Young):			
Cond. prob. of spousal NE transition	6.55%	9.34%	27.02%
Cond. prob. of spousal NU transition	1.96%	6.94%	2.01%
Cond. prob. of spousal NN transition	91.49%	83.72%	70.97 %
IV. Excluding Disabled/Ill (Old):			
Cond. prob. of spousal NE transition	4.17%	3.42%	8.53%
Cond. prob. of spousal NU transition	0.88%	2.77%	0.50%
Cond. prob. of spousal NN transition	94.95%	93.81%	90.97%
V. Excluding Retired and Disabled/Ill (Young):			
Cond. prob. of spousal NE transition	6.55%	9.36%	27.23%
Cond. prob. of spousal NU transition	1.97%	6.96%	2.05%
Cond. prob. of spousal NN transition	91.48%	83.68%	70.72%
VI. Excluding Retired and Disabled/Ill (Old):			
Cond. prob. of spousal NE transition	4.74%	3.62%	11.20%
Cond. prob. of spousal NU transition	1.16%	3.40%	0.89%
Cond. prob. of spousal NN transition	94.11%	92.99%	87.91%

Notes: This table shows the probability of a spousal transition from out of the labor force conditional on primary earner transitions by reasons for non-participation.

B.1.6 Business Cycle

Table B.7 Joint Labor Market Transitions

	Primary earner transition		
	EE	EU	EN
NBER Recession, Young			
Cond. prob. of spousal NE transition	6.48%	7.74%	22.38%
Cond. prob. of spousal NU transition	1.98%	8.73%	0.99%
Cond. prob. of spousal NN transition	91.55%	83.53%	76.63%
NBER Recession, Old			
Cond. prob. of spousal NE transition	4.14%	5.43%	7.71%
Cond. prob. of spousal NU transition	0.83%	2.76%	0.68%
Cond. prob. of spousal NN transition	95.03%	91.81%	91.61%
No NBER Recession, Young			
Cond. prob. of spousal NE transition	6.68%	9.53%	27.45%
Cond. prob. of spousal NU transition	2.00%	6.63%	2.14%
Cond. prob. of spousal NN transition	91.31%	83.85%	70.41%
No NBER Recession, Old			
Cond. prob. of spousal NE transition	4.30%	3.46%	8.80%
Cond. prob. of spousal NU transition	0.91%	2.75%	0.54%
Cond. prob. of spousal NN transition	94.79%	93.79%	90.66%

Notes: This table shows the probability of a spousal transition from out of the labor force conditional on primary earner transitions by state of the business cycle.

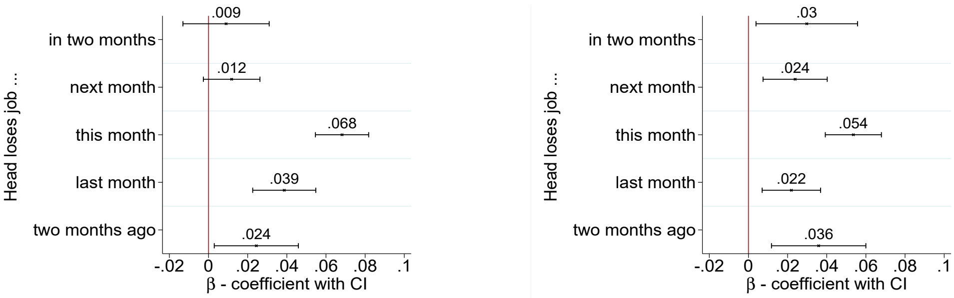
B.1.7 Income

Table B.8 Joint Labor Market Transitions by Past Income

	Primary earner transition		
	EE	EU	EN
I. Low Income (All):			
Cond. prob. of spousal NE transition	5.57%	7.41%	15.89%
Cond. prob. of spousal NU transition	2.98%	5.81%	1.64%
Cond. prob. of spousal NN transition	92.45%	86.79%	82.48%
II. High Income (All):			
Cond. prob. of spousal NE transition	5.91%	8.93%	20.71%
Cond. prob. of spousal NU transition	1.14%	4.75%	0.73%
Cond. prob. of spousal NN transition	92.95%	86.32%	78.55%
III. Low Income (Young):			
Cond. prob. of spousal NE transition	6.22%	8.66%	23.30%
Cond. prob. of spousal NU transition	2.37%	7.48%	2.38%
Cond. prob. of spousal NN transition	91.41%	83.85%	74.32%
IV. Low Income (Old):			
Cond. prob. of spousal NE transition	3.66%	3.52%	8.11%
Cond. prob. of spousal NU transition	0.95%	2.41%	0.66%
Cond. prob. of spousal NN transition	95.39%	94.08%	91.24%
V. High Income (Young):			
Cond. prob. of spousal NE transition	7.24%	7.13%	40.17%
Cond. prob. of spousal NU transition	1.19%	4.18%	0.12%
Cond. prob. of spousal NN transition	91.57%	88.69%	59.71%
VI. High Income (Old):			
Cond. prob. of spousal NE transition	4.76%	3.66%	11.121%
Cond. prob. of spousal NU transition	0.90%	2.84%	0.49%
Cond. prob. of spousal NN transition	94.34%	93.50%	88.30%

Notes: This table shows the probability of a spousal transition from out of the labor force conditional on primary earner transitions by income.

B.1.8 Dynamics Response for Other Age Groups



(a) Age 36 to 45

(b) Age 46 to 55

Figure B.1 Δ Pr(Spouse enters LF) this month

Notes: Figure B.1 shows the change in probability that a non-participating spouse enters the labor force (either as unemployed or as employed) if the household head loses/lost the job in two months, next month, this month, last month or two months ago, respectively, relative to the baseline in which the household head remains employed. The sample includes couples in which one spouse is working and one spouse is out of the labor force between age 36 and 45 (Figure B.1a) and between age 46 and 55 (Figure B.1b) from the Current Population Survey (CPS), waves 1994 until 2020. Age refers to the non-participating spouse. The regression producing the coefficients is Equation 2.1.

Appendix C

Appendix to Chapter 3

C.1 Data Appendix

We use data provided by the Congressional Budget Office (CBO) to compute average tax and transfer rates. Given our focus on the working population we restrict ourselves to non-elderly households.

As income concept we use a broad measure of market income including wages, employee's contribution for deferred compensation, employer's contribution for health insurance, employer's share of payroll taxes, federal unemployment tax, corporate tax borne by labor, corporate tax borne by capital, capital gains, tax-exempt interest, taxable interest, positive rent, dividends, and other market income.

For taxes, we consider all taxes the CBO reports. This includes only federal taxes. The specific components are individual income taxes, payroll taxes, corporate income taxes, and excise taxes. Tax credits such as the Earned Income Tax Credit (EITC) and the Child Tax Credit (CTC) are included in taxes in the data. While these credits are at least partially refundable and therefore could be considered transfers, we cannot separate them in the CBO data and therefore leave them in taxes.

For transfers, we only consider transfers that are meant to provide income security. Because our model is an infinite horizon model we do not consider transfers to the elderly such as social security and Medicare. Also, we do not model health shocks or disability, so we leave Medicaid and Supplemental Security Income (SSI) out of our transfer measure. Our transfer measure includes programs to provide income security such as the Supplemental Nutrition Assistance Program (SNAP; commonly known as food stamps), Temporary Assistance for Needy Families (TANF), housing assistance, and some other smaller means-tested transfer programs.

C.2 Model Appendix

C.2.1 Analytical Model: First Best with a Representative Agent

As a special case of the analytical model we consider a representative agent version. Using this simple framework we show that there is optimally a negative relationship between the size of transfers and the progressivity of the income tax code, even absent redistributive concerns.

First Best. To find the first best, we maximize utility subject to the resource constraint of the economy. The problem reads

$$\max_{c,n} \log c - B\frac{n^{1+\varphi}}{1+\varphi} \text{ s.t. } c = n - G. \tag{C.1}$$

The first order condition implies

$$Bn^{\varphi}(n-G) = 1. \tag{C.2}$$

Implementation with Loglinear Taxes. Consider now a government that has access to only the loglinear income tax function, but not a lump sum transfer. The problem of the representative agent is

$$\max_{c,n} \log c - B\frac{n^{1+\varphi}}{1+\varphi} \text{ s.t. } c = \lambda n^{1-\tau}. \tag{C.3}$$

The first order condition of this problem implies labor supply, as stated in the main text:

$$n = \left(\frac{1-\tau}{B}\right)^{\frac{1}{1+\varphi}}. \tag{C.4}$$

In the main text we claim that

$$\tau = -\frac{G}{n-G} \tag{C.5}$$

implements the first best allocation. To see this, plug equation (C.5) into equation (C.4):

$$\begin{aligned}
n &= \left(\frac{1+\frac{G}{n-G}}{B}\right)^{\frac{1}{1+\varphi}} \\
\Rightarrow B n^{1+\varphi} &= \frac{G}{n-G}+1 \\
\Rightarrow B n^{1+\varphi} &= \frac{n}{n-G} \\
\Rightarrow B n^{\varphi}(n-G) &= 1,
\end{aligned} \tag{C.6}$$

which is the condition describing labor supply in the first best, equation (C.2). Hence, according to equation (C.5) the first best is implemented with just the loglinear tax function with a progressivity of zero if there is no exogenous spending requirement and with a negative progressivity if required spending is positive.

Implementation with Taxes and Transfers. When the government uses taxes and transfers, the household problem becomes

$$\max_{c,n} \log c - B\frac{n^{1+\varphi}}{1+\varphi} \text{ s.t. } c = \lambda n^{1-\tau} + T. \tag{C.7}$$

The first order condition in this case is

$$Bn^{\varphi} = \frac{\lambda(1-\tau)n^{-\tau}}{\lambda n^{1-\tau}+T}. \tag{C.8}$$

The government budget constraint reads

$$n - \lambda n^{1-\tau} = G + T. \tag{C.9}$$

We can use the government budget constraint to rewrite the household first order condition as

$$Bn^{1+\varphi}(n-G) = (1-\tau)(n-G-T). \tag{C.10}$$

The claim in the main text is that the first best is implemented with taxes and transfers whose relationship is determined by

$$\tau = -\frac{G+T}{n-G-T}. \tag{C.11}$$

To see this, we again plug in equation (C.11) into equation (C.10) and recover the first order condition describing the first best allocation:

$$\begin{aligned}
&Bn^{1+\varphi}(n-G) = \left(1+\frac{G+T}{n-G-T}\right)(n-G-T) \\
\Rightarrow &Bn^{1+\varphi}(n-G) = \left(\frac{n}{n-G-T}\right)(n-G-T) \\
\Rightarrow &Bn^{\varphi}(n-G) = 1.
\end{aligned} \tag{C.12}$$

Hence, in the representative agent version of the model in which the first best is implemented with the loglinear tax function and a lump sum transfer, the optimal progressivity τ is a strictly decreasing function of the size of the transfer T.

C.2.2 Analytical Model: Welfare without Transfers

In this section we derive the closed form expression for welfare when the government's only available tool is the loglinear tax function.

Idiosyncratic Productivity. For convenience, we summarize again the properties of the process for idiosyncratic productivity $z_{i,t}$. We assume $\log z_{i,t} = \alpha_{i,t}$ and $\alpha_{i,t} = \rho_z \alpha_{i,t-1} + \omega_{i,t}$ with $\omega_{i,t} \sim N(-\frac{v_\omega}{2(1+\rho_z)}, v_\omega)$ and $\alpha_{i,0} \sim N(-\frac{v_\omega}{2(1-\rho_z^2)}, \frac{v_\omega}{1-\rho_z^2})$. We can rewrite $\alpha_{i,t}$ as:

$$\alpha_{i,t} = \rho_z \alpha_{i,t-1} + \omega_{i,t} = \rho_z^t \alpha_{i,0} + \sum_{j=0}^{t-1} \rho_z^j \omega_{t-j}. \tag{C.13}$$

Taking the expectation, we get by linearity of the expectations operator:

$$\mathbb{E}[\alpha_{i,t}] = \rho_z^t \mathbb{E}[\alpha_{i,0}] + \sum_{j=0}^{t-1} \rho_z^j \mathbb{E}[\omega_{t-j}] = -\rho_z^t \frac{v_\omega}{2(1-\rho_z^2)} - \frac{1-\rho_z^t}{1-\rho_z} \frac{v_\omega}{2(1+\rho_z)} = -\frac{v_\omega}{2(1-\rho_z^2)}. \tag{C.14}$$

For the variance, we obtain:

$$\mathbb{V}[\alpha_{i,t}] = \rho_z^{2t} \mathbb{V}[\alpha_{i,0}] + \sum_{j=0}^{t-1} \rho_z^{2j} \mathbb{V}[\omega_{t-j}] = \rho_z^{2t} \frac{v_\omega}{1-\rho_z^2} + \frac{1-\rho_z^{2t}}{1-\rho_z^2} v_\omega = \frac{v_\omega}{1-\rho_z^2}. \tag{C.15}$$

And as $z = \exp(\alpha)$, $\mathbb{E}[z_{i,t}] = 1 \ \forall t$.

Household Problem. The consumers solve a static problem:

$$\max_{\{c_{it}, n_{it}\}} \log c_{it} - \frac{B}{1+\varphi} (n_{it})^{1+\varphi} \quad \text{s.t.} \quad c_{it} = \lambda [\exp(\alpha_{it}) n_{it}]^{1-\tau} \tag{C.16}$$

Computing the first order condition we get:

$$n_{it}=\left(\frac{1-\tau}{B}\right)^{\frac{1}{1+\varphi}}\equiv n_0. \tag{C.17}$$

Computing λ. The budget constraint of the government is

$$G=\int_0^1\left(y_i-\lambda y_i^{1-\tau}\right)di. \tag{C.18}$$

Output is given by

$$Y_t=\int_0^1 y_{i,t}d_i=\int_0^1\exp\left(\alpha_{i,t}\right)n_{i,t}d_i=n_0\int_0^1\exp\left(\alpha_{i,t}\right)d_i=n_0. \tag{C.19}$$

We also need

$$\begin{aligned}\tilde{Y}_t&=\int y_{i,t}^{1-\tau}dF_\alpha^t=\int\left[n_0\exp(\alpha_{i,t})\right]^{1-\tau}dF_\alpha^t=n_0^{1-\tau}\int\exp(\alpha_{i,t}(1-\tau))dF_\alpha^t\\&=n_0^{1-\tau}\exp\left(-\tau\frac{(1-\tau)v_\omega}{2(1-\rho_z^2)}\right).\end{aligned} \tag{C.20}$$

Therefore, we can express λ as:

$$\lambda=\frac{Y-G}{\tilde{Y}}=\frac{n_0-G}{n_0^{1-\tau}}\exp\left(\tau\frac{(1-\tau)v_\omega}{2(1-\rho_z^2)}\right) \tag{C.21}$$

Welfare. We can now compute welfare in closed form. We plug the equilibrium value of consumption and hours worked into the utility function:

$$\begin{aligned}u_i(c,n)&=\log\lambda+(1-\tau)\log\left(\exp(\alpha_{i,t})n_0\right)-\frac{B}{1+\varphi}n_0^{1+\varphi}\\&=\log\lambda+\frac{1-\tau}{1+\varphi}\log\left(\frac{1-\tau}{B}\right)+(1-\tau)\alpha_{i,t}-\frac{1-\tau}{1+\varphi}\end{aligned} \tag{C.22}$$

We now integrate over the distribution of households:

$$\begin{aligned}W(\tau)&=\int u_i(c,n)dF_\alpha^t\\&=\int[\log\lambda+\frac{1-\tau}{1+\varphi}\log\left(\frac{1-\tau}{B}\right)+(1-\tau)\alpha_{i,t}-\frac{1-\tau}{1+\varphi}]dF_\alpha^t\\&=\log\lambda+\frac{1-\tau}{1+\varphi}\log\left(\frac{1-\tau}{B}\right)-\frac{(1-\tau)v_\omega}{2(1-\rho_z^2)}-\frac{1-\tau}{1+\varphi}\end{aligned} \tag{C.23}$$

We can now plug in the closed-form solution for λ.

$$
\begin{aligned}
W(\tau) &= \log\lambda + \frac{1-\tau}{1+\varphi}\log\left(\frac{1-\tau}{B}\right) - \frac{(1-\tau)v_\omega}{2(1-\rho_z^2)} - \frac{1-\tau}{1+\varphi} \\
&= \log(n_0 - G) - (1-\tau)\log(n_0) + \tau\frac{(1-\tau)v_\omega}{2(1-\rho_z^2)} + \frac{1-\tau}{1+\varphi}\log\left(\frac{1-\tau}{B}\right) - \frac{(1-\tau)v_\omega}{2(1-\rho_z^2)} - \frac{1-\tau}{1+\varphi} \\
&= \log\left(\left(\frac{1-\tau}{B}\right)^{\frac{1}{1+\varphi}} - G\right) - \frac{1-\tau}{1+\varphi}\log\left(\frac{1-\tau}{B}\right) + \tau\frac{(1-\tau)v_\omega}{2(1-\rho_z^2)} + \frac{1-\tau}{1+\varphi}\log\left(\frac{1-\tau}{B}\right) - \frac{(1-\tau)v_\omega}{2(1-\rho_z^2)} - \frac{1-\tau}{1+\varphi}
\end{aligned}
\tag{C.24}
$$

And therefore, we obtain equation (3.9) in the main text:

$$
W(\tau) = \log\left(\left(\frac{1-\tau}{B}\right)^{\frac{1}{1+\varphi}} - G\right) - \frac{1-\tau}{1+\varphi} - \frac{(1-\tau)^2 v_\omega}{2(1-\rho_z^2)}. \tag{C.25}
$$

C.2.3 Analytical Model: Welfare with Transfers

We now derive welfare as a function of progressivity τ and the transfer T. The logic of the derivation is the same as in the previous section. However, given that we cannot express everything in closed form, we are going to linearize around the case of a zero transfer.

Household Problem. Households now solve the following static problem:

$$
\max_{\{c_{it}, n_{it}\}} \log c_{it} - B\frac{n_{it}^{1+\varphi}}{1+\varphi} \quad \text{s.t.} \quad c_{it} = \lambda[\exp(\alpha_{it})n_{it}]^{1-\tau} + T. \tag{C.26}
$$

The first-order condition reads:

$$
Bn_{it}^{\varphi} = \frac{1}{c_{it}}(\exp(\alpha_{it})n_{it})^{-\tau}\exp(\alpha_{it})\lambda(1-\tau). \tag{C.27}
$$

That is, after rearranging:

$$
Bn_{it}^{1+\varphi} + \frac{T}{\lambda}Bn_{it}^{\varphi+\tau}\exp(-(1-\tau)\alpha_{it}) - (1-\tau) = 0. \tag{C.28}
$$

From equation (C.28) let us define the function $G(T, n_{it})$ s.t. $G(T, n_{it}) = 0$: at the optimum the labor decision is such that, for a given T, $G(T, n_{it}(T)) = 0$.

Linear Approximation of Labor Policy. At $T = 0 \equiv T_0$, we know that $n_{it}(T_0) = \left(\frac{1-\tau}{B}\right)^{\frac{1}{1+\varphi}} \equiv n_0$ and $\lambda_0 = \frac{n_0 - G}{n_0^{1-\tau}}\eta^{\tau/2}$ with $\eta = \exp\left((1-\tau)\frac{v_\omega}{1-\rho_z^2}\right)$. The implicit function theorem holds, and we can compute the slope of n_{it} in the neighborhood of T_0 as:

$$\frac{\partial n_{it}(T)}{\partial T}\Big|_{(T_0,n_0,\lambda_0)} = -\frac{\frac{\partial G(T,n_{it})}{\partial T}\big|_{(T_0,n_0,\lambda_0)}}{\frac{\partial G(T,n_{it})}{\partial n_{it}}\big|_{(T_0,n_0,\lambda_0)}}, \tag{C.29}$$

where the two partial derivatives are:

$$\frac{\partial G(T,n_{it})}{\partial T}\Big|_{(T_0,n_0,\lambda_0)} = \frac{n_0^{1-\tau}}{n_0-G}\eta^{-\frac{\tau}{2}}\exp\left[-(1-\tau)\,\alpha\right]Bn_0^{\varphi+\tau} \tag{C.30}$$

and

$$\frac{\partial G(T,n_{it})}{\partial n_{it}}\Big|_{(T_0,n_0,\lambda_0)} = B(1+\varphi)\,n_0^{\varphi}. \tag{C.31}$$

We obtain a linear approximation around (T_0,n_0,λ_0) of $n_{it}(T)$ denoted $\hat{n}_{it}(T)$:

$$\begin{aligned}\hat{n}_{it}(T) &= n_0 + T\frac{\partial n_{it}(T)}{\partial T}\Big|_{(T_0,h_0,\lambda_0)} \\ &= n_0 - \frac{T}{1+\varphi}\frac{n_0}{n_0-G}\eta^{-\frac{\tau}{2}}\exp(-(1-\tau)\alpha_{it}).\end{aligned} \tag{C.32}$$

Computing λ. We compute λ again from the budget constraint of the government, which is

$$G+T = \int_0^1 \left(y_i - \lambda y_i^{1-\tau}\right) di. \tag{C.33}$$

Hence, we can compute λ as

$$\lambda(T) = \frac{Y-G-T}{\tilde{Y}}, \tag{C.34}$$

so that we need to compute again Y and $\tilde{Y}$. We start with Y:

$$\begin{aligned}Y_t &= \int_0^1 y_{i,t}d_i = \int_0^1 \exp(\alpha_{i,t})n_{i,t}d_i \\ &= n_0\int_0^1 \exp(\alpha_{i,t})di - \frac{T}{1+\varphi}\frac{n_0}{n_0-G}\eta^{-\frac{\tau}{2}}\int_0^1 \exp(\tau\alpha_{it})di \\ &= n_0 - \frac{T}{1+\varphi}\frac{n_0}{n_0-G}\eta^{-\tau}.\end{aligned} \tag{C.35}$$

To obtain $\tilde{Y}$ we first approximate $\hat{n}^{1-\tau}$ around the case of a zero transfer. Using equation (C.32) we get

$$\hat{n}_{it}^{1-\tau}(T) = \left[n_0 - \frac{T}{1+\varphi}\frac{n_0}{n_0-G}\eta^{-\frac{\tau}{2}}\exp(-(1-\tau)\alpha_{it})\right]^{1-\tau}, \tag{C.36}$$

which we can linearize to obtain

$$\hat{n}_{it}^{1-\tau}(T) = n_0^{1-\tau} - \frac{T}{1+\varphi}(1-\tau)\frac{n_0^{1-\tau}}{n_0-G}\eta^{-\frac{\tau}{2}}\exp(-(1-\tau)\alpha_{it}). \tag{C.37}$$

It follows that

$$\begin{aligned}
\tilde{Y}_t &= \int_0^1 \left[n_0^{1-\tau} - \frac{T}{1+\varphi}(1-\tau)\frac{n_0^{1-\tau}}{n_0-G}\eta^{-\frac{\tau}{2}}\exp(-(1-\tau)\alpha_{it})\right]\exp\left[(1-\tau)\alpha_{it}\right]di \\
&= n_0^{1-\tau}\eta^{-\frac{\tau}{2}} - \frac{T}{1+\varphi}(1-\tau)\frac{n_0^{1-\tau}}{n_0-G}\eta^{-\frac{\tau}{2}} \\
&= n_0^{1-\tau}\eta^{-\frac{\tau}{2}}\left[1 - \frac{T}{1+\varphi}\frac{1-\tau}{n_o-G}\right].
\end{aligned} \tag{C.38}$$

Using these expressions we linearize λ around the case of a zero transfer:

$$\lambda(T) = \frac{n_0 - \frac{T}{1+\varphi}\frac{n_0}{n_0-G}\eta^{-\tau} - G - T}{n_0^{1-\tau}\eta^{-\frac{\tau}{2}}\left[1 - \frac{T}{1+\varphi}\frac{1-\tau}{n_o-G}\right]} \tag{C.39}$$

and

$$\lambda'(T)\Big|_{(T_0)} = \frac{1}{1+\varphi}\frac{1}{\eta^{-\frac{\tau}{2}}n_0^{1-\tau}}\left[-\frac{n_0}{n_0-G}\eta^{-\tau} - (\varphi+\tau)\right] \tag{C.40}$$

such that

$$\hat{\lambda}(T) = \lambda_0 + \frac{T}{1+\varphi}\frac{1}{\eta^{-\frac{\tau}{2}}n_0^{1-\tau}}\left[-\frac{n_0}{n_0-G}\eta^{-\tau} - (\varphi+\tau)\right]. \tag{C.41}$$

Welfare. Finally, we approximate utility around a zero transfer. The utility of an individual agent is given by

$$\begin{aligned}
u_{it}(c_{it}, n_{it}) &= \log c_{it} - \frac{B}{1+\varphi}n_{it}^{1+\varphi} \\
&= \log\left[\lambda(T)\left[\exp(\alpha)n_{it}\right]^{1-\tau} + T\right] - \frac{B}{1+\varphi}n_{it}^{1+\varphi} \\
&= \log\left[\lambda(T)\left[\exp\left[(1-\tau)\alpha_{it}\right]n_0^{1-\tau} - \frac{T}{1+\varphi}(1-\tau)\frac{n_0^{1-\tau}}{n_0-G}\eta^{-\frac{\tau}{2}}\right] + T\right] - \frac{B}{1+\varphi}n_{it}^{1+\varphi},
\end{aligned} \tag{C.42}$$

which can be approximated as

$$\hat{u} = u_0 + T\left\{ \frac{\lambda'(T)\exp\left[(1-\tau)\,\alpha_{it}\right] n_0^{1-\tau} - \lambda_0 \frac{1}{1+\varphi}\,(1-\tau)\,\frac{n_0^{1-\tau}}{n_0-G}\eta^{-\frac{\tau}{2}} + 1}{\lambda_0 \exp\left[(1-\tau)\,\alpha_{it}\right] n_0^{1-\tau}} \right.$$
$$\left. + \frac{B}{1+\varphi} n_0^{\varphi} \frac{n_0}{n_0-G}\eta^{-\frac{\tau}{2}} \exp\left[-(1-\tau)\,\alpha_{it}\right] \right\}$$
$$= u_0 + T\left\{ \frac{1}{1+\varphi}\frac{1}{n_0-G}\left[-\frac{n_0}{n_0-G}\eta^{-\tau} - (\varphi+\tau)\right] - \frac{1}{1+\varphi}\frac{1-\tau}{n_0-G}\eta^{-\frac{\tau}{2}}\exp\left[-(1-\tau)\,\alpha_{it}\right] + \frac{1}{n_0-G}\eta^{-\frac{\tau}{2}}\exp\left[-(1-\tau)\,\alpha_{it}\right] \right.$$
$$\left. + \frac{B}{1+\varphi} n_0^{\varphi} \frac{n_0}{n_0-G}\eta^{-\frac{\tau}{2}} \exp\left[-(1-\tau)\,\alpha_{it}\right] \right\} \tag{C.43}$$

Integrating this yields

$$W(\tau,T) = W(\tau,0) + T\left\{ \frac{1}{1+\varphi}\frac{1}{n_0-G}\left[-\frac{n_0}{n_0-G}\eta^{-\tau} - (\varphi+\tau)\right] - \frac{1}{1+\varphi}\frac{1-\tau}{n_0-G}\eta^{1-\tau} + \eta^{1-\tau}\frac{1}{n_0-G} \right.$$
$$\left. + (1-\tau)\,\frac{1}{1+\varphi}\frac{1}{n_0-G}\eta^{1-\tau} \right\}$$
$$= W(\tau,0) + \frac{T}{1+\varphi}\frac{\eta^{-\tau}}{n_0(\tau)-G}\left(-\frac{n_0(\tau)}{n_0(\tau)-G} + (1-\tau)\eta + (\varphi+\tau)\,(\eta-\eta^{\tau}) \right). \tag{C.44}$$

Appendix D

Appendix to Chapter 4

D.1 Partial Equilibrium Model Appendix

D.1.1 Non-Homothetic CES

We specify preferences over the three commodities using a non-homothetic CES utility function as introduced by Hanoch (1975) and recently popularized in the structural change literature by Comin, Lashkari, and Mestieri (2021). These preferences do not admit expressing Marshallian demand functions in closed form, so we use the Hicksian demand function (4.5) and the expenditure function (4.4). These can be derived from the expenditure minimization problem of a household subject to constraint (4.3)

$$\min \sum_{j=1}^{J} p_j C_j - \chi \left[\sum_{j=1}^{J} (\Omega_j C^{\varepsilon_j})^{\frac{1}{\sigma}} C_j^{\frac{\sigma-1}{\sigma}} - 1 \right], \tag{D.1}$$

where χ is the Lagrange multiplier. The first order conditions with respect to C_j read

$$p_j - \chi (\Omega_j C^{\varepsilon_j})^{\frac{1}{\sigma}} \frac{\sigma - 1}{\sigma} C_j^{-\frac{1}{\sigma}} \quad \forall\, j. \tag{D.2}$$

Dividing the first order conditions for two commodities we obtain

$$\frac{p_j}{p_i} = \frac{(\Omega_j C^{\varepsilon_j})^{\frac{1}{\sigma}}}{(\Omega_i C^{\varepsilon_i})^{\frac{1}{\sigma}}} \frac{C_j}{C_i}. \tag{D.3}$$

Using this we can express any commodity j as a function of commodity i:

$$C_j = \left(\frac{p_i}{p_j} \right)^{\sigma} \frac{\Omega_j C^{\varepsilon_j}}{\Omega_i C^{\varepsilon_i}} C_i. \tag{D.4}$$

We can plug this back into the constraint (4.3) for every good j such that we are left with one equation for a generic commodity i. Solving this equation yields the Hicksian demand function. Multiplying the expression with the respective price and summing over all commodities yields the expenditure function.

Below we are going to need the first and second derivatives of the expenditure function, which are given by

$$E'(C) = \frac{1}{1-\sigma}\left[\sum_{j=1}^{I}\Omega_j\varepsilon_j C^{\varepsilon_j-1}p_j^{1-\sigma}\right]E(C)^{\sigma}, \tag{D.5}$$

$$E''(C) = \frac{\sigma}{1-\sigma}\left[\sum_{j=1}^{I}\Omega_j\varepsilon_j C^{\varepsilon_j-1}p_j^{1-\sigma}\right]E(C)^{\sigma-1}E'(C) + \frac{E(C)^{\sigma}}{1-\sigma}\left[\sum_{j=1}^{J}\Omega_j\varepsilon_j(\varepsilon_j-1)C^{\varepsilon_j-2}p_j^{1-\sigma}\right]. \tag{D.6}$$

D.1.2 Derivation of Optimal Tax Formula

We derive the optimal tax formula using standard perturbation techniques following Saez (2001). The idea is that at the optimal tax schedule a small perturbation of the tax system may not affect welfare. To derive the optimal tax formula, we therefore consider a small increase of the marginal tax rate, $d\mathscr{T}'$ in the interval $[y^*-dy, y^*]$.

The first effect of this perturbation of the tax schedule is that individuals earning incomes larger than y^* pay $d\mathscr{T}'dy$ more taxes. This reduces their welfare, which must be weighted with their endogenous marginal social welfare weight, but gives the government additional funds. Hence, the mechanical effect can be written as

$$dW^M(\theta^*) = d\mathscr{T}'dy\int_{\theta^*}^{\bar{\theta}}\left(1-\frac{\frac{u'(\theta)}{E'(\theta)}w(\theta)}{\lambda}\right)dF(\theta). \tag{D.7}$$

The second effect is the substitution effect. Individuals whose income falls in the interval where the marginal tax rate changes adjust their labor supply by

$$\frac{\partial y(\theta^*)}{\partial\mathscr{T}'}d\mathscr{T}' = -\varepsilon_{y,1-\mathscr{T}'}(\theta^*)\frac{y(\theta^*)}{1-\mathscr{T}'(y(\theta^*))}d\mathscr{T}'. \tag{D.8}$$

This adjustment has to be weighted with the mass of individuals who are affected:

$$h(y(\theta^*))dy = f(\theta^*)\frac{1}{\varepsilon_{y,\theta}(\theta^*)}\frac{\theta^*}{y(\theta^*)}dy. \tag{D.9}$$

The impact of this change on individuals' welfare is of second order by the envelope theorem. However, the labor supply change has a first order impact on the government budget. Hence, the substitution effect can be written as follows:

$$dW^{S}(\theta^{*}) = -\mathscr{T}'(y(\theta^{*}))\,\varepsilon_{y,1-\mathscr{T}'}(\theta^{*})\,\frac{y(\theta^{*})}{1-\mathscr{T}'(y(\theta^{*}))}d\mathscr{T}'f(\theta^{*})\,\frac{1}{\varepsilon_{y,\theta}(\theta^{*})}\frac{\theta^{*}}{y(\theta^{*})}dy. \tag{D.10}$$

The third effect is an income effect. Given that we use preferences with income effects, labor supply of those earnings more than $y(\theta^{*})$ adjust their labor supply even though their marginal tax rates do not change. We denote the income effect as $\eta(\theta) = \frac{\partial y(\theta)}{\partial T}$. The income effect can be written as

$$dW^{I}(\theta^{*}) = d\mathscr{T}'dy\int_{\theta^{*}}^{\bar{\theta}}\eta(\theta)\,\mathscr{T}'(y(\theta))\,dF(\theta). \tag{D.11}$$

At the optimum it has to be the case that

$$dW^{M} + dW^{S} + dW^{I} = 0. \tag{D.12}$$

Solving this equation yields the optimal tax formula

$$\frac{\mathscr{T}'(y(\theta^{*}))}{1-\mathscr{T}'(y(\theta^{*}))} = \frac{\varepsilon_{y,\theta}(\theta^{*})}{\varepsilon_{y,1-\mathscr{T}'}(\theta^{*})}\frac{\int_{\theta^{*}}^{\bar{\theta}}\left[1-\frac{\frac{u'(\theta)}{E'(\theta)}w(\theta)}{\lambda}+\eta(\theta)\,\mathscr{T}'(y(\theta))\right]dF(\theta)}{f(\theta^{*})\,\theta^{*}}. \tag{D.13}$$

From the first order condition for the optimal choice of the lump sum element we can obtain an expression for λ:

$$\int_{\underline{\theta}}^{\overline{\theta}}\left(1-\frac{\frac{u'(\theta)}{E'(\theta)}w(\theta)}{\lambda}+\eta(\theta)\mathscr{T}'(y(\theta))\right)dF(\theta) = 0 \leftrightarrow \lambda = \frac{\int_{\underline{\theta}}^{\overline{\theta}}\frac{u'(\theta)}{E'(\theta)}w((\theta))dF(\theta)}{\int_{\underline{\theta}}^{\overline{\theta}}(1+\eta(\theta)\mathscr{T}'(y(\theta)))\,dF(\theta)}. \tag{D.14}$$

To apply the formula, we still need expressions for $\varepsilon_{y,\theta}(\theta)$, $\varepsilon_{y,1-\mathscr{T}'}(\theta)$, and $\eta(\theta)$. To derive these, consider the household problem

$$\max_{C,y}\; u(C) - v\left(\frac{y}{\theta}\right)\; s.t.\, E(C) = y - \mathscr{T}(y) + T. \tag{D.15}$$

The first order condition of this problem reads as

$$v'\left(\frac{y}{\theta}\right) - \frac{u'(C)}{E'(C)}(1-\mathscr{T}'(y))\theta = 0. \tag{D.16}$$

Applying the implicit function theorem to the first order condition, we obtain

$$\frac{\partial y}{\partial(1-\mathscr{T}')} = -\frac{-\frac{\theta u'(C)}{E'(C)} + \frac{(E''(C)u'(C)-u''(C)E'(C))(1-\mathscr{T}')\theta}{(E'(C))^2}\frac{\partial C}{\partial(1-\mathscr{T}')}}{v''\left(\frac{y}{\theta}\right)\frac{1}{\theta} + \mathscr{T}''(y)\,\theta\frac{u'(C)}{E'(C)} + \frac{(E''(C)u'(C)-u''(C)E'(C))(1-\mathscr{T}')\theta}{(E'(C))^2}\frac{\partial C}{\partial y}}. \tag{D.17}$$

From the budget constraint we get

$$\frac{\partial C}{\partial(1-\mathscr{T}')} = 0 \tag{D.18}$$

$$\frac{\partial C}{\partial y} = \frac{1-\mathscr{T}'(y)}{E'(C)}. \tag{D.19}$$

Hence, it follows for the elasticity that

$$\varepsilon_{y,1-\mathscr{T}'}(\theta) = \frac{\frac{\theta u'(C)}{E'(C)}(1-\mathscr{T}'(y))}{\left[v''\left(\frac{y}{\theta}\right)\frac{1}{\theta} + \mathscr{T}''(y)\,\theta\frac{u'(C)}{E'(C)} + \frac{(E''(C)u'(C)-u''(C)E'(C))(1-\mathscr{T}')\theta}{(E'(C))^2}\frac{1-\mathscr{T}'(y)}{E'(C)}\right]y} \tag{D.20}$$

Similarly, we can compute

$$\frac{\partial y}{\partial\theta} = -\frac{-v''\left(\frac{y}{\theta}\right)\frac{y}{\theta^2} - \frac{u'(C)}{E'(C)}(1-\mathscr{T}') + \frac{(E''(C)u'(C)-u''(C)E'(C))(1-\mathscr{T}')\theta}{(E'(C))^2}\frac{\partial C}{\partial\theta}}{v''\left(\frac{y}{\theta}\right)\frac{1}{\theta} + \mathscr{T}''(y)\,\theta\frac{u'(C)}{E'(C)} + \frac{(E''(C)u'(C)-u''(C)E'(C))(1-\mathscr{T}')\theta}{(E'(C))^2}\frac{\partial C}{\partial y}} \tag{D.21}$$

From the budget constraint we get that

$$\frac{\partial C}{\partial\theta} = 0, \tag{D.22}$$

so that we obtain

$$\varepsilon_{y,\theta} = \frac{v''\left(\frac{y}{\theta}\right)\frac{y}{\theta} + \theta\frac{u'(C)}{E'(C)}(1-\mathscr{T}')}{\left[v''\left(\frac{y}{\theta}\right)\frac{1}{\theta} + \mathscr{T}''(y)\,\theta\frac{u'(C)}{E'(C)} + \frac{(E''(C)u'(C)-u''(C)E'(C))(1-\mathscr{T}')\theta}{(E'(C))^2}\frac{1-\mathscr{T}'(y)}{E'(C)}\right]y}. \tag{D.23}$$

Combining these results, we obtain

$$\begin{aligned}\frac{\varepsilon_{y,\theta}(\theta)}{\varepsilon_{y,1-\mathscr{T}'}(\theta)} &= \frac{v''\left(\frac{y}{\theta}\right)\frac{y}{\theta} + \theta\frac{u'(C)}{E'(C)}(1-\mathscr{T}')}{\frac{\theta u'(C)}{E'(C)}(1-\mathscr{T}'(y))}\\ &= \frac{v''\left(\frac{y}{\theta}\right)\frac{y}{\theta} + v'\left(\frac{y}{\theta}\right)}{v'\left(\frac{y}{\theta}\right)}\\ &= 1 + \frac{1}{\varepsilon}\end{aligned} \tag{D.24}$$

Finally, we can compute the income effect as

$$\eta(\theta) = -\frac{\frac{(E''(C)u'(C)-u''(C)E'(C))(1-\mathscr{T}')\theta}{(E'(C))^2}\frac{\partial C}{\partial T}}{v''\left(\frac{y}{\theta}\right)\frac{1}{\theta}+\mathscr{T}''(y)\,\theta\frac{u'(C)}{E'(C)}+\frac{(E''(C)u'(C)-u''(C)E'(C))(1-\mathscr{T}')\theta}{(E'(C))^2}\frac{\partial C}{\partial y}} \tag{D.25}$$

with

$$\frac{\partial C}{\partial b} = \frac{1}{E'(C)}. \tag{D.26}$$

Rearranging this equation yields

$$\eta(\theta) = -\frac{\frac{(E''(C)u'(C)-u''(C)E'(C))\theta}{(E'(C))^2}}{\frac{u'(C)}{\varepsilon_{\frac{y}{\theta}}}+\frac{\mathscr{T}''(y)}{1-\mathscr{T}'(y)}\theta u'(C)+\frac{(E''(C)u'(C)-u''(C)E'(C))(1-\mathscr{T}')\theta}{(E'(C))^2}}. \tag{D.27}$$

D.2 General Equilibrium Model Appendix

D.2.1 Firm Problem

In this section we derive equation (4.15) from the firm problem (4.14). The first order conditions of the problem are

$$p_jA_j\alpha_jH_j^{\frac{\rho-1}{\rho}-1}\left[\alpha_jH_j^{\frac{\rho-1}{\rho}}+(1-\alpha_j)L_j^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}-1} = w_H, \tag{D.28}$$

$$p_jA_j(1-\alpha_j)L_j^{\frac{\rho-1}{\rho}-1}\left[\alpha_jH_j^{\frac{\rho-1}{\rho}}+(1-\alpha_j)L_j^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}-1} = 1. \tag{D.29}$$

Dividing equation (D.28) by equation (D.29) yields

$$w_H = \frac{\alpha_j}{1-\alpha_j}\left(\frac{H_j}{L_j}\right)^{-\frac{1}{\rho}}.$$

This can be solved for the demand for low-skilled labor as a function of the skill premium and high-skilled labor demand:

$$L_j = H_jw_H^{\rho}\left(\frac{1-\alpha_j}{\alpha_j}\right)^{\rho}. \tag{D.30}$$

We can plug (D.30) into (D.28) to obtain

$$
\begin{aligned}
w_H &= p_j A_j \alpha_j H_j^{\frac{\rho-1}{\rho}-1} \left[\alpha_j H_j^{\frac{\rho-1}{\rho}} + (1-\alpha_j) \left(H_j w_H^{\rho} \left(\frac{1-\alpha_j}{\alpha_j} \right)^{\rho} \right)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}-1} \\
\Rightarrow w_H &= p_j A_j \alpha_j H_j^{\frac{-1}{\rho}} \left[H_j^{\frac{\rho-1}{\rho}} \left[\alpha_j + (1-\alpha_j) w_H^{\rho-1} \left(\frac{1-\alpha_j}{\alpha_j} \right)^{\rho-1} \right] \right]^{\frac{1}{\rho-1}} \\
\Rightarrow w_H &= p_j A_j \alpha_j H_j^{\frac{-1}{\rho}} H_j^{\frac{1}{\rho}} \left[\alpha_j + (1-\alpha_j) w_H^{\rho-1} \left(\frac{1-\alpha_j}{\alpha_j} \right)^{\rho-1} \right]^{\frac{1}{\rho-1}} \\
\Rightarrow w_H^{\rho-1} &= p_j^{\rho-1} A_j^{\rho-1} \alpha_j^{\rho-1} \left[\alpha_j + (1-\alpha_j) w_H^{\rho-1} \left(\frac{1-\alpha_j}{\alpha_j} \right)^{\rho-1} \right] \\
\Rightarrow w_H^{\rho-1} &= p_j^{\rho-1} A_j^{\rho-1} \alpha_j^{\rho} + p_j^{\rho-1} A_j^{\rho-1} \alpha_j^{\rho-1} (1-\alpha_j) w_H^{\rho-1} \left(\frac{1-\alpha_j}{\alpha_j} \right)^{\rho-1} \\
\Rightarrow w_H^{\rho-1} &= p_j^{\rho-1} A_j^{\rho-1} \alpha_j^{\rho} + p_j^{\rho-1} A_j^{\rho-1} (1-\alpha_j)^{\rho} w_H^{\rho-1} \\
\Rightarrow 1 &= p_j^{\rho-1} \frac{A_j^{\rho-1}}{w_H^{\rho-1}} \alpha_j^{\rho} + p_j^{\rho-1} A_j^{\rho-1} (1-\alpha_j)^{\rho} \\
\Rightarrow 1 &= p_j^{\rho-1} A_j^{\rho-1} \left[\frac{\alpha_j^{\rho}}{w_H^{\rho-1}} + (1-\alpha_j)^{\rho} \right] \\
\Rightarrow 1 &= p_j A_j \left[\frac{\alpha_j^{\rho}}{w_H^{\rho-1}} + (1-\alpha_j)^{\rho} \right]^{\frac{1}{\rho-1}} .
\end{aligned}
$$

Solving this equation for p_j delivers equation (4.15) in the main text.

D.2.2 Equilibrium

To solve for an equilibrium in the market for high-skilled labor, we first compute demand for high-skilled labor. For that purpose, we use equations (4.13), (D.30), and (4.15). First, plug (D.30) into (4.13):

$$
\begin{aligned}
Y_j &= A_j \left[\alpha_j H_j^{\frac{\rho-1}{\rho}} + (1-\alpha_j) \left(H_j w_H^{\rho} \left(\frac{1-\alpha_j}{\alpha_j} \right)^{\rho} \right)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}} \\
\Rightarrow Y_j &= A_j \left[\alpha_j H_j^{\frac{\rho-1}{\rho}} + (1-\alpha_j) H_j^{\frac{\rho-1}{\rho}} \left(\frac{\alpha_j}{1-\alpha_j} \frac{1}{w_H} \right)^{1-\rho} \right]^{\frac{\rho}{\rho-1}} \\
\Rightarrow Y_j &= A_j H_j \left[\alpha_j + (1-\alpha_j) \left(\frac{\alpha_j}{1-\alpha_j} \frac{1}{w_H} \right)^{1-\rho} \right]^{\frac{\rho}{\rho-1}} \\
\Rightarrow Y_j &= A_j H_j \left[\alpha_j + (1-\alpha_j)^{\rho} \alpha_j^{1-\rho} w_H^{\rho-1} \right]^{\frac{\rho}{\rho-1}}
\end{aligned}
$$

$$\Rightarrow \left(\frac{Y_j}{A_j H_j}\right)^{\frac{\rho-1}{\rho}} = \alpha_j + (1-\alpha_j)^{\rho}\,\alpha_j^{1-\rho} w_H^{\rho-1}$$

$$\Rightarrow \frac{\alpha_j^{\rho}}{w_H^{\rho-1}} + (1-\alpha_j)^{\rho} = \left(\frac{Y_j}{A_j H_j}\right)^{\frac{\rho-1}{\rho}} \frac{1}{\alpha_j^{1-\rho}} \frac{1}{w_H^{\rho-1}}.$$

Next, we can rewrite (4.15) to obtain

$$\frac{\alpha_j^{\rho}}{w_H^{\rho-1}} + (1-\alpha_j)^{\rho} = (p_j A_j)^{1-\rho}.$$

Combining the last two equations yields

$$\left(\frac{Y_j}{A_j H_j}\right)^{\frac{\rho-1}{\rho}} \frac{1}{\alpha_j^{1-\rho}} \frac{1}{w_H^{\rho-1}} = (p_j A_j)^{1-\rho}$$

$$\Rightarrow \left(\frac{Y_j}{A_j H_j}\right)^{\frac{\rho-1}{\rho}} = \left(\frac{\alpha_j p_j A_j}{w_H}\right)^{1-\rho}$$

$$\frac{Y_j}{A_j H_j} = \left(\frac{\alpha_j p_j A_j}{w_H}\right)^{-\rho}.$$

Rearranging this gives equation (4.18).

D.2.3 Computation

We can solve for the equilibrium as follows.

1. Guess the wage premium w_H.
2. Compute the prices implied by the guess for the wage premium using equation (4.15).
3. Guess a lump-sum transfer.
4. Solve household problem for all worker types.
   (a) Guess a consumption aggregator C.
   (b) Compute the expenditure function and its derivative, as given by equations (4.4) and (D.5).
   (c) Guess labor supply n.
   (d) Check whether the first order condition (4.12) holds. If yes, move on; if no, update guess for n.
   (e) Check whether the budget constraint (4.9) holds. If yes, move on; if no, update guess for C.
5. Check whether the government budget clears (4.17). If yes, move on; if no, update guess for T.

6. Check whether the high-skilled labor market (4.20) clears. If yes, we have found the equilibrium; if no, update guess for w_H.

D.2.4 Alternative Pareto Weights

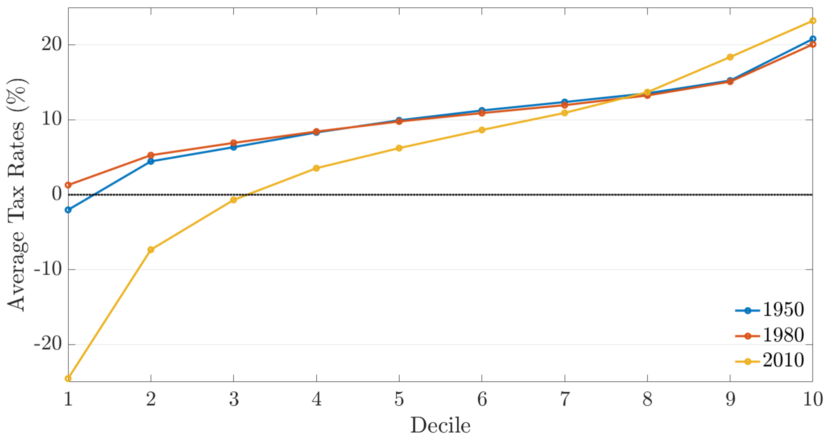


Figure D.1 Optimal Average Tax Rates: U.S.

Notes: This figure shows optimal average rates given by the entire tax-and-transfer system by income decile for the U.S. in 1950, 1980, and 2010. Pareto weights are kept constant across years as described in the main text.

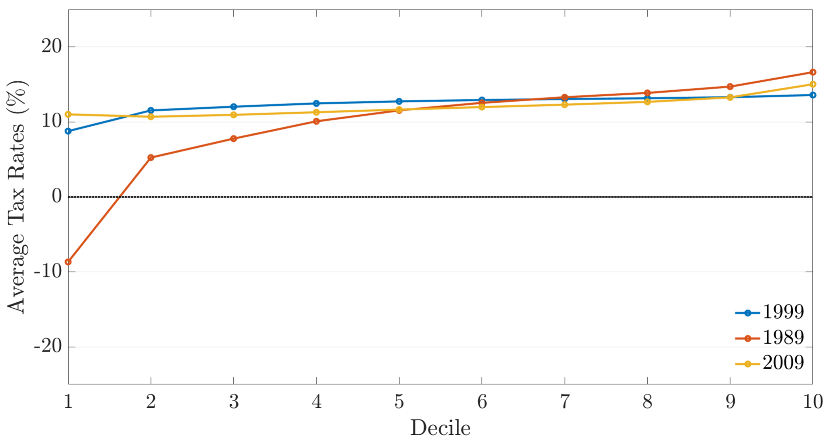


Figure D.2 Optimal Average Tax Rates: China

Notes: This figure shows optimal average rates given by the entire tax-and-transfer system by income decile for China in 1989, 1999, and 2009. Pareto weights are kept constant across years as described in the main text.